بسم الله الرحمن الرحيم

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بقسم التوثيق الإلكتروني بمركز الشبكات وتكنولوجيا المعلومات دون أدنى

مسئولية عن محتوى هذه الرسالة.

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ON THE OSCILLATION OF SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS

FORWARDED FOR THE pH. D. DEGREE IN SCIENCE
ORDINARY DIFFERENTIAL EQUATIONS
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بسم الله الرحمن الرحيم

« وقل رب زدني علماً »

« صدق الله العظيم »

To

My Parents

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Summary

The thesis is devoted to the study of oscillatory and asymptotic behavior of solutions of some general forms of nonlinear differential equations which contain some neutral delayed on the form

$$\frac{d}{dt}\left[x(t)+P(t)\,F(x(g_1(t)))\right]+G(t,x(g_2(t)))-H(t,x(g_3(t)))=0 \quad (1)$$

$$\frac{d^2}{dt^2}\left[x(t)+P(t)\,F(x(g_1(t)))\right]+G(t,x(g_2(t)))-H(t,x(g_3(t)))=0 \quad (2)$$

$$\frac{d^2}{dt^2}\left[x(t)+P(t)g(x(h(t)))\right]-\int_0^{\sigma(t)}F(t,x(q(s)))d\mu(t,s)=0 \quad (3)$$

The obtained results improve and extend some of the known results in the literature. Some sufficient conditions are relaxed. we give also an example arising in mathematical ecology differential equation of the form

$$\frac{d}{dt}\left[x(t)+P(t)x(t-\tau)\right]+r(t)\,(1+x(t))f(x(t-\sigma))=0. \quad (4)$$

Introduction

The aim of this thesis is to discuss oscillatory and asymptotic behavior of eventually solutions of Neutral Delayed Differential Equations (NDDE).

Definition (1)

A solution of differential equation is oscillatory if the set of its zeros is unbounded from above, or otherwise it is nonoscillatory.

A differential equation is oscillatory if all its solutions are oscillatory, otherwise it is nonoscillatory. For example the differential equation

$$x'' + x = 0, \qquad t \geq 0$$

is oscillatory since its general solutions are oscillatory while the equation

$$x''' - x'' + x' - x = 0$$

is nonoscillatory because one of its nontrivial solutions, namely, e^t, is not oscillatory.

Definition (2)

A neutral delayed differential equation (NDDE) is that in which the highest order derivative of the unknown function appears in the equation both with and without delays. For example, the differential equation:

$$\frac{d}{dt}\left[x(t) \pm P(t)x(t-\tau)\right] + Q(t)x(t-\sigma) = 0,$$

where $P, Q \in C[[t_0, \infty), R))$, *and* $\tau, \sigma \in [0, \infty)$ is a first order netural delayed differential equation.

Definition (3)

The function f is said to be eventually enjoy a property K if there exists $t_0>0$, such that for $t \geq t_0$ the function enjoys the property K.

Chapter 1, contains several basic lemmas from analysis that are used in several occasions throughout this monograph.

In 1994 Lu [20] obtained several new existence theorems for nonoscillatory solutions of the equation:

$$\frac{d}{dt}\left[x(t) - \sum_{i=1}^{k} c_i(t) \;\; x(t - v_i(t))\right] + Q(t)x(t-\sigma) = 0 \quad , \qquad (1)$$

where

$$Q(t), c_j(t) \in [t_0, \infty), Q(t), c_j(t) \geq 0, \;\; 0 < v_0 < v_j(t) \leq v, \;\; \sigma \geq 0, \; j \in I_k = \{1, 2, ..., k\}$$

In section 1.3, we discuss the (NDDE) of the type

$$\frac{d}{dt}\left[x(t) + \sum_{i=1}^{n} a_i \; x(t-\tau_i) - \sum_{j=1}^{m} b_j \; x(t-\rho_j)\right] = -\sum_{k=1}^{w} q_k \; x(t-\sigma_k), \qquad (2)$$

where

$a_i, \tau_i > 0,\quad for\ i \in I = \{1,2,\ldots,n\}\quad and\ b_j, \rho_j > 0,\quad for\ j \in J = \{1,2,\ldots,m\},\quad and$ $q_k, \sigma_k > 0,\quad for\ k \in K = \{1,2,\ldots,w\}.$

,

Our results generalize some of the results of Lu [20]. In 1994, Erbe and Kong [6(b)] showed that the first order (NDDE):

$$\frac{d}{dt}\left[x(t) - P\,x(t-\tau)\right] + \sum_{i=1}^{n} q_i(t)\,x((t-\sigma_i) = 0\ , \qquad (3)$$

where

$$q_i(t) \in C([t_0,\infty),[0,\infty)),\ P \in [0,1],\quad \tau, \sigma_i \in (0,\infty),\quad i = 1,2,\ldots,n$$

is oscillatory if:

$$\lim_{t\longrightarrow\infty} \inf \left[\rho e^{\mu\tau} + \frac{1}{l\mu} \sum_{i=1}^{n} e^{\mu\sigma_i} \int_t^{t+l} q_i(s)ds \right] > 1$$

For

$$l \in \{\tau, \sigma_1, \sigma_2, \ldots, \sigma_n\} \qquad \mu > 0.$$

In section 1.3, we show that equation (2) is oscillatory if:

$$\min_l \left[\frac{1}{l\mu} \sum_{k=0}^{w} e^{\mu\sigma_k} q_k + \sum_{j=1}^{m} b_j e^{\mu\rho_j} - \sum_{i=1}^{n} a_i e^{\mu\tau_i} \right] > 1$$

For

$$l \in \{\tau_1, \tau_2, \ldots, \tau_n, \rho_1, \rho_2, \ldots, \rho_m, \sigma_1, \sigma_2, \ldots, \sigma_w\},\quad \mu > 0.$$

Grammatikopoulos, Ladas and Meimoridou [11(a)], [11(d)] discussed the second order (NDDE):

$$\frac{d^2}{d^2t}\left[x(t)+P(t)\,x(t-\tau)\right]+Q(t)x(t-\sigma)=0 \tag{4}$$

Where

$$P,Q\in C([t_0,\infty),R),$$

and the delays τ and σ are nonnegative real numbers. They showed that every unbounded solution of (4) oscillates if the conditions:

A1) $Q(t)\geq 0$ for all $t\geq t_0$

A2) $-1\leq P(t)\leq 0$ for all $t\geq t_0$

A3) $\int_{t_0}^{\infty}Q(s)ds=\infty$

Hold. Moreover it had been shown that the derivative of every differentiable solution of (4) oscillates if (A3) holds and $P(t)=P\geq 0$. In section 1.4, we show that all unbounded solutions of the second order (NDDE) of the type

$$\frac{d^2}{d^2t}\left[x(t)-\sum_{i=1}^{n}P_i(t)x(t-\tau_i(t))\right]+\sum_{j=1}^{m}q_j(t)x(t-\sigma_j(t))=0 \tag{5}$$

Where

$$\tau_i,P_i,q_j\in[t_0,\infty),\sigma_j\in C^1[t_0,\infty),P_i\geq 0,q_j>0,0<\tau_0<\tau_i(t)\leq\tau,$$
$$0<\sigma_0<\sigma_j(t)\leq\sigma, and\, \sigma_j'(t)<0\, for all, 1\leq i\leq n, 1\leq j\leq m.$$

are oscillatory, if the two conditions

1) $\sum_{i=1}^{n}P_i(t)\leq 1$, and

2) $\int_{t_0}^{\infty}\sum_{j=1}^{m} q_j(s)ds = \infty$

are satisfied. Moreover, we show that the derivatives of all differentiable solutions of equation (5) are oscillatory if (1) and (2) hold.

In 1991, Greaf, Grammatikopoules and Spikes [10(d)], [10(c)] discussed asymptotic behavior of oscillatory and nonoscillatory solutions of the two first order nonlinear NDDES.:

$$\frac{d}{dt}\left[x(t) + P(t)x(t-\tau)\right] \pm Q(t)f(x(t-\sigma)) = 0, \tag{6}$$

where

$$P, Q : [t_0, \infty) \longrightarrow R$$

Are continuous with neither P nor Q identically zero on any half line $[t_0, \infty)$, τ and σ are nonnegative constants, and $f : R \longrightarrow R$ is continuous.

In section 2.2, we obtain two oscillation theorems for the differential equation:

$$\frac{d}{dt}\left[x(t) + P(t)\, f(x(g_1(t)))\right] + G(t, x(g_2(t))) - H(t, x(g_3(t))) = 0, \tag{7}$$

where all the mentioned functions are continuous with $uf(u) > 0$ for $u \neq 0$, $g_i(t) \leq t$, for i=1,2,3, and $G, H: [t_0, \infty) \times R \longrightarrow R$ with $G \neq H$ on R.

The obtained theorems improve and extend some of the results of [11(b)] . Moreover, in section 2.3, we establish sufficient conditions for all solutions to be nonoscillatory. In particular, our results include the work of [10(c)].

In 1995, Li[19] studied the oscillatory behavior of solutions of the linear differential equation

$$[a(t)\,x'(t)]' + P(t)\,x(t) = 0 \ , \tag{8}$$

where

$$a(t) \in C^1([t_0,\infty);(0,\infty)), \text{ and } P(t) \in C([t_0,\infty),R), \ \ t_0 \geq 0$$

The main result of Li [19] is given by the following theorem.

Theorem 1 [19]

Let

$$D_0 = \{(t,s): t > s \geq t_0\}, \text{ and } \quad D = \{(t,s): t \geq s \geq t_0\}.$$

Let $H \in C(D,R)$ satisfy the following two conditions:

(a) $H = (t,t) = 0$ for $t \geq t_0$, $H(t,s) > 0$, *for* $t > s \geq t_0$

(b) H has a continuous and non positive partial derivative on D_0. Suppose that $h: D \longrightarrow R$ is a continuous function with

$$-\frac{\partial H}{\partial s}(t,s) = h(t,s)\sqrt{H(t,s)} \quad \text{for all} \quad (t,s) \in D_0$$

If there exists a function $f \in C^1[t_0,\infty)$ such that: