



شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكرو فيلم

بسم الله الرحمن الرحيم



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جامعة عين شمس

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Mathematical Treatments for Some Game Issues

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SUMMARY

We, humans, cannot survive without interacting with other humans, and ironically, it sometimes seems that we have survived despite those interactions. The subject matter of game theory is exactly those interactions within a group of individuals (or governments, firms, etc.) where the actions of each individual have an effect on the outcome that is of interest to all. In this thesis, we will study the concept of surreal numbers and their relationship to some games and link them in an understandable way, and then put some concepts related to the relationship of the graph to the games and study a game represented on the graphs and we show some properties.

This dissertation falls into four chapters as following.

Chapter One: In this chapter, we will present a critical introduction to basic concepts of game theory. These include basic definitions of simultaneous game and sequential game, describing strategic games, discussion the combinatorial games in detail and explain some models of combinatorial games.

Chapter Two: In this chapter we study construction of the Surreal Numbers, showing it is a class that forms the totally ordered field, and then explore some of new numbers, we present to the reader some algebraic operations related to combinatorial games and gives a detailed outlook of the Surreal Numbers. A fresh outlook to some combinatorial mathematical algebraic operations, through the evaluation of a deduced several algebraic concepts. The findings in this chapter have been published in International Journal of Scientific & Engineering Research in 2020 under the name “Maan T. Alabdullah, Essam El-Seidy and Neveen S. Morcos (2020). On Numbers and Games, International Journal of Scientific & Engineering Research, Volume 11, Issue 2, February - 2020.”.

Chapter Three: We begin our investigation of the surreals by looking at what numbers are created on the first few days, and then verifying some basic properties of all numbers. Then we add the addition and multiplication of the surreal numbers, we end the chapter by introducing how surreal numbers can be used to analyze games, in particular the game of Hackenbush

Chapter Four: In this chapter, we provide basic knowledge of definitions and concepts related to the concept of matching in the graph. We are studying a model of games based on two players who take turns adding edges to G , this process eventually produces a maximal matching of the graph. We call the first Maximizer and second player Minimizer. The first aims to get a final matching to be large while the second one wants to reduce it. Maximizer wins if he manages a maximal matching while Minimizer wins if he can prevent him from doing this. The matcher number $\alpha_g(G)$ is the number of edges chosen when both players play optimally, while the matching number $\alpha(G)$ is the number of maximum matching edges. In this chapter, we study the relationship between $\alpha_g(G)$ and $\alpha(G)$. And we also prove some results on types of graph.

Chapter 1

An Introduction to Game Theory

1.1 Introduction

Game theory is the study of mathematical models of strategic interaction between rational decision-makers [1]. It has applications in all fields of social science, as well as in logic and computer science. Originally, it addressed zero-sum games, in which one person's gains result in losses for the other participants [2]. Today, game theory applies to a wide range of behavioral relations and is now an umbrella term for the science of logical decision making in humans, animals, and computers.

Modern game theory began with the idea regarding the existence of mixed-strategy equilibria in two-person zero-sum games and its proof by John von Neumann [1]. Von Neumann's original proof used the Brouwer fixed-point theorem on continuous mappings into compact convex sets, which became a standard method in game theory and mathematical economics [3]. His paper was followed by the 1944 book *Theory of Games and Economic Behavior*, co-written with Oskar Morgenstern, which considered cooperative games of several players. The second edition of this book provided an axiomatic theory of expected utility, which allowed mathematical statisticians and economists to treat decision-making under uncertainty [3].

Game theory was developed extensively in the 1950s by many scholars. It was later explicitly applied to biology in the 1970s, although similar developments go back at least as far as the 1930s. Game theory has been widely recognized as an important tool in many fields. As of 2014, with the Nobel Memorial Prize in Economic Sciences going to game theorist Jean Tirole, eleven game theorists have won the economics Nobel Prize. John Maynard Smith was awarded the Crafoord Prize for his application of game theory to biology [1].

It as often as possible happens that you are confronted with making a choice or choose a best strategy from a few decisions. For example, you may need to choose whether to put resources into stocks or bonds, or you may need to pick a hostile play to use in a football game. In both of these examples, the outcome relies on something you can't control. In the first example, your success partly depends on the future behavior of the economy. In the second example, it depends on the defensive strategy chosen by the opposing team. We can model situations like these using game theory [2].

Game theory is a study with the principle motivation behind discovering a response to the question: how to respond in both conflict and cooperation situations, as well as combined ones. This idea pre-characterizes a condition that there must be no less than two sides of a connection towards one another to discuss conflict/cooperation [4].

Game theory is a well -developed field of study has pulled in a portion of the world's most prominent mathematicians, won two Nobel Prizes and is even credited with winning the Cold War. The roots of game theory go far back in time. Late work recommends that the division of a legacy depicted in the Talmud (in the early years of the first thousand years) predicts the modern theory of cooperative games and, in 1713, James Waldegrave wrote out a strategy for a card game that provided the first known solution for a two-player game [1].

Regardless of these early efforts, the book *The Theory of Games and Economic Behavior* by Von Neumann and Oskar Morgenstern (published in 1944) is typically credited as the beginning of the formal investigation of game theory. This spearheading work concentrated on finding unique strategies that allowed players to minimize their maximum losses (minimax solution) by considering, for every possible strategy of their own, all the possible responses of other players. Expanding upon von Neumann's prior work on two player games where the winnings of one player are equal and contrary to the losses of his opponent (zero-sum) and where each player knows the strategies available to all players and their consequences (perfect information), von Neumann and Morgenstern

extended the minimax theorem to include games involving imperfect information and games with more than two players [2].

The brilliant time of game theory happened in the 1950s and 1960s when scientists concentrated on finding sets of strategies, known as equilibria, to “solve” a game if all players behaved rationally. The most well-known of these is the Nash Equilibrium proposed by John Nash, later made celebrated in the film "A Beautiful Mind" featuring Russell Crowe. A Nash equilibrium exists if no player can singularly move to enhance his own outcome. In other words, they have no impetus to change, since their strategies are all the better they can do given the activities of alternate players. Nash additionally made critical commitments to bargaining theory and examined cooperative games where dangers and guarantees are completely tying and enforceable. In 1965, Reinhard Selten introduced the concept of sub-game perfect equilibria, which describes strategies that deliver a Nash equilibrium across every sequential sub-game of the original game [5].

John Harsanyi formalized Nash’s work and developed incomplete information games in 1967. He, alongside John Nash and Reinhard Selten, won the Nobel Prize for Economics in 1994. Another vital commitment to game theory during the 1950s and 1960s was Luce and Raiffa's book, *Games and Decisions*. The Prisoner's Dilemma, presented by the RAND Partnership and exceptionally well known to any MBA understudy, is additionally a result of this period.

Further adding to the praise of game theory, another Nobel Prize was granted to game theorists, Robert Aumann and Thomas Schelling, in 2005. Schelling used game theory in his 1960 book *The Strategy of Conflict* to clarify why credible threats of nuclear annihilation from the U.S. and the former Soviet Union were counterbalancing through mutually assured destruction and therefore were not likely to be used. Aumann's work was mathematical and centered around whether cooperation expands if games are continually repeated rather than played out in a single encounter. He demonstrated that collaboration is more outlandish when there are numerous members, at the point when communications are occasional,

when the time skyline is short or when others' activities can't be plainly watched [2].

These days game theory is identified with different areas, for example, ecology and biology, in particular related to evolution. In these areas, the individual's behavior does not rely on rationality, but on different perspectives, for example, fitness. Consistently, game theory has been connected to a wide range of fields of study, including artificial intelligence, bargaining, political science and real-world business decisions [3,4].

1.2 Simultaneous game

Rock–paper–scissors is an example of a simultaneous game. In game theory, a simultaneous game is a game where each player chooses his action without knowledge of the actions chosen by other players [6]. Simultaneous games contrast with sequential games, which are played by the players taking turns (moves alternate between players). Normal form representations are usually used for simultaneous games

1.3 Sequential game

In game theory, a sequential game is a game where one player chooses their action before the others choose theirs [6]. Importantly, the later players must have some information of the first's choice, otherwise the difference in time would have no strategic effect. Sequential games hence are governed by the time axis and represented in the form of decision trees.

Unlike sequential games, simultaneous games do not have a time axis as players choose their moves without being sure of the other's and are usually represented in the form of payoff matrices. Extensive form representations are usually used for sequential games, since they explicitly illustrate the sequential aspects of a game. Combinatorial games are usually sequential games.

Games such as chess, infinite chess, backgammon, tic-tac-toe and Go are examples of sequential games. The size of the decision trees can vary according to game complexity, ranging from the small game tree of tic-tac-toe, to an immensely complex game tree of chess so large that even computers cannot map it completely [7]. In sequential games with perfect information, a subgame perfect equilibrium can be found by backward induction.

Game theory is the intelligent examination of situations of conflict and cooperation. More specifically, a game is defined to be any situation in which [8]:

- The rules of the game are Specific, comprehensible and known to all players.
- The players follow these rules.
- There are between two and an infinite number of players.
- The game consists of moves performed at the same time (simultaneous game), or one at a time (sequential game); the number of moves is finite.
- Each player has a set of strategies (finite or infinite). In theory, a game is restricted to choose between strategies by each player. A choice of strategy can be a probability of selection among the subsets of strategies. What it means is that the player and their opponents do not know which strategy they will choose; this will be dictated by the chance.
- After the game ends, each of the players gets a certain result, its value being numerical and described by means of payoff functions.

1.4 Describing Strategic Games

To have the capacity to apply game theory, a first step is to define the boundaries of the strategic game under thought. Games are defined in terms of their rules. The rules of a game incorporate information about the