بسم الله الرحمن الرحيم

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تم رفع هذه الرسالة بواسطة / سامية زكى يوسف

بقسم التوثيق الإلكتروني بمركز الشبكات وتكنولوجيا المعلومات دون أدنى مسئولية عن محتوى هذه الرسالة.

ملاحظات: لا يوجد

Faculty of Education
Department of Mathematics

Spectral Solutions of Differential Equations via Some New Classes of Orthogonal Polynomials and Special Functions

Presented by

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A Thesis Submitted
to
Faculty of Education
In partial Fulfillment of the Requirements for
the Degree of Ph.D. in Teacher's Preparation of Science
(Pure Mathematics)
Department of Mathematics
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2022

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Thesis Title: Spectral Solutions of Differential Equations via Some New Classes of Orthogonal Polynomials and Special Functions

Degree: Ph.D. in Teacher's Preparation of Science in Pure Mathematics

(Numerical Analysis)

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Abstract

In this thesis, we propose new efficient spectral techniques for handling certain types of partial differential equations and partial fractional differential equations such as; the one-dimensional linear hyperbolic partial differential equation of the first-order, the fractional diffusion wave equation, the heat conduction equation, the fractional initial-value problem, the time-fractional partial differential problem, the telegraph equation, and the nonlinear fractional Rayleigh-Stokes equation. In these techniques, we employ new basis functions of the shifted Chebyshev polynomials of the fifth and sixth kinds. The key idea of the presented techniques depends on transforming these equations with their underlying conditions into systems of algebraic equations in the unknown expansion coefficients. Our studies are supported by a careful convergence analysis of the suggested shifted fifth and sixth kinds Chebyshev expansions. Finally, some numerical examples are presented to confirm the accuracy and efficiency of the proposed techniques. We want to mention that this thesis consists of seven chapters and all the results of chapter two up to chapter seven are completely new. Some results obtained in this thesis are already published in four international prestigious journals with high impact factors and some others are submitted for publication and still under referring.

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List of Abbreviations

AE............. Absolute error
MAE........... Maximum absolute error
HPDEs........ Hyperbolic first-order partial differential equations
FDWE......... Fractional diffusion-wave equation
FDWED...... Fractional diffusion wave equation with damping
FIVP............ Fractional initial value problem
FRSE........... Fractional Rayleigh-Stokes equation
CPU............ Computational time
$C_i(x)$........... The shifted Chebyshev polynomials of the fifth-kind
$Y_i^*(x)$............The shifted Chebyshev polynomials of the sixth-kind

Acknowledgements

First of all, my gratitude and thanks to gracious **Allah** who always helps and guides me. I would like to thank **the prophet Mohamed** "peace be upon him" who urges us to seek knowledge and who is the teacher of mankind.

I would like also to thank the supervision committee:

Prof. Dr. Galal Mahrous Moatimid, Professor of Applied Mathematics, Faculty of Education , Ain Shams University, for accepting supervision of me, learning me ethics and scientific research assets, his good care for me and for his continuous support to reach the best.

Dr. Youssri Hassan Youssri, Associate Professor of Pure Mathematics, Faculty of Science, Cairo University, for suggesting the topic of the thesis, who provided me with guidance and continuous encouragement. He offered me much of his precious time and provided me with his wisdom and knowledge through many discussions we had.

Prof. Dr. Waleed Mohamed Abd-Elhameed, Professor of Pure Mathematics, Faculty of Science, Cairo University, who provided me with guidance and continuous encouragement. He did his best for the success of this work through precious comments, valuable reviews and remarks. His passion and extraordinary dedication to work have always inspired me and encouraged me to work harder.

Also, I would like to thank **Prof. Dr. Ehab Fathy Abd-Elfattah** and the head of Mathematics Department **Prof. Dr. Mohamed Yahia Abouzeid** for providing me with all facilities required for the success of this work.