



بسم الله الرحمن الرحيم

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Ain Shams University
Faculty of Engineering
Engineering Physics and Mathematics Department

Wave solutions of nonlinear evolution equations modeling physical phenomena

By

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B.Sc. Electrical Engineering, Ain Shams University, 2013
M.Sc. Engineering Mathematics, Ain Shams University, 2019

**A Thesis Submitted in Partial Fulfillment of the Requirement for the Degree of
Doctor of philosophy in Applied Engineering Mathematics**

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Cairo 2022



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STATEMENT

This thesis is submitted as partial fulfillment of Ph.D. degree in Applied Engineering Mathematics, Faculty of Engineering, Ain Shams University.

The author carried out the work included in this thesis, and no part of it has been submitted for a degree or qualification at any other scientific entity.

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Thesis Summary

This thesis aims to present some aspects of a few methods that have been introduced recently to solve nonlinear partial differential equations (PDEs) which represent some physically relevant systems in applied physics, applied mathematics, especially in optics and fluids.

This thesis is organized in twelve chapters as listed below:

Chapter-1 is introductory and consists of four sections. In first section, the importance of nonlinear partial differential equation in modeling many physical phenomena is discussed. Some different types of traveling wave solutions are introduced in the second section. In third section, we study some properties of wave phenomena. In Fourth section, we review some important methods for solving nonlinear partial differential equations.

In Chapter-2, we utilize the modified Jacobi elliptic function method and the modified (G'/G) expansion method to get explicit exact solutions for the fourth order non-linear partial differential equation that describe the pulses dynamics in the optical fibers. With the aid of these methods, we get many exact solutions like bright and dark solitons, periodic, hyperbolic, and rational type solutions.

In Chapter-3, the improved modified extended tanh function method is applied to secure optical soliton solutions in magneto-optic waveguides with parabolic non-local law of refractive index. Dark, bright and singular solitons are obtained. Also, periodic wave solutions, Jacobi elliptic function solutions, Weierstrass elliptic solutions, exponential solutions, hyperbolic solutions and other solutions are extracted.

In Chapter-4, the improved modified extended tanh scheme is implemented to extract exact travelling wave solutions to cubic–quartic perturbed nonlinear Schrödinger's equation with sextic-power law of refractive index. Various types of solutions are extracted such as bright soliton,

singular soliton, dark soliton, singular periodic wave solution, periodic wave solution, Jacobi elliptic functions, plane wave and hyperbolic wave solutions.

In Chapter-5, the improved modified extended tanh function method is applied to study the perturbed Gerdjikov-Ivanov equation (PGI) which describe the dynamics of the soliton in optical fibers. New exact solutions for the PGI equation will be introduced including weierstrass elliptic, Jacobi elliptic, dark and bright solitons, singular periodic and rational type solutions. The graphical representation of some solutions is illustrated.

In Chapter-6, we study the coupled Fokas-Lenelss system which describes the wave dynamics in optical fibers. Studying is conducted with the aid of the improved modified extended tanh method. Many types of solutions are obtained such as dark solitons, bright solitons, singular solitons, periodic wave solutions, singular periodic wave solutions, hyperbolic wave solutions, plane wave solutions, Jacobi elliptic wave solutions and Weierstrass elliptic wave solutions. furthermore, 3D and contour plots of some obtained solutions are introduced to show their physical nature.

In Chapter-7, the improved modified extended tanh approach is implemented to investigate the strain wave model which governs the wave propagation in micro-structured solids. Solitary and other wave solutions are provided for this model such as singular, dark, and bright solitary type solutions. In addition, periodic, Jacobi elliptic, hyperbolic, exponential and Weierstrass elliptic type solutions are constructed.

In Chapter-8, the improved modified extended tanh approach is used to analyze sixth-order dispersive non-linear Schrödinger's equation with kerr law nonlinearity. Abundant solutions are derived by the proposed method such as dark, bright, dark-bright and singular solitary waves, as well as hyperbolic and periodic function solutions are generated. our results are bolstered by introducing 3D visualizations of some solutions.

In Chapter-9, the modified direct algebraic method is applied for the perturbed nonlinear Schrödinger equation (NLSE) describing the dynamics of optical solitons in metamaterials, in the presence of quadratic-cubic nonlinearity. Various types of solutions are extracted such as bright solitons, singular solitons, dark solitons, singular periodic wave solutions, periodic wave solutions, plane wave solutions, Jacobi elliptic wave solutions, exponential wave solutions, hyperbolic wave solutions and Weierstrass elliptic solutions. Moreover, for the physical illustration of the obtained solutions, 3D graphs are presented.

In Chapter-10, The solitons in birefringent optical fibers with Hamiltonian perturbations and Kerr law nonlinearity is studied in this chapter. By using the modified extended direct algebraic method, we obtained soliton solutions and other solutions for complex Ginzburg-Landau (CGL) equation having kerr law nonlinearity with Hamiltonian terms. Bright solitons, dark solitons, singular solitons, Weierstrass elliptic, plane wave, hyperbolic, exponential, singular periodic and periodic wave solutions are extracted.

In Chapter-11, we study a new (3+1) dimensional Boiti–Leon–Manna–Pempinelli equation that describes the wave dynamics in an incompressible fluid. We employ an improved simple equation method to extract explicit exact wave solutions with distinct physical structures. Dark soliton solutions, singular solitons, and singular periodic wave solutions are extracted.

In Chapter-12, the modified extended mapping method is applied to obtain exact solutions for the Gilson-Pickering equation which describes the wave propagation in plasma physics to secure many types of solutions such as shock wave, singular waves, periodic waves, singular periodic waves, Jacobi elliptic functions, plane waves, hyperbolic waves, Weierstrass elliptic solutions and exponential type solutions.

Keywords: Bright soliton, Dark Soliton, Exact solutions, nonlinear partial differential equations

List of publications

1. I. Samir, N. Badra, H. M. Ahmed, A. H. Arnous, and A. S. Ghanem, Solitary wave solutions and other solutions for Gilson-Pickering equation by using the modified extended mapping method, Results in Physics. (2022) 105427. (Impact factor: 4.476)
2. I. Samir, N. Badra, H. M. Ahmed, and A. H. Arnous, Solitons in birefringent fibers for CGL equation with Hamiltonian perturbations and Kerr law nonlinearity using modified extended direct algebraic method, Communications in Nonlinear Science and Numerical Simulation. 102 (2021) 105945. (Impact factor: 4.260)
3. I. Samir, N. Badra, H. M. Ahmed, and A. H. Arnous, Optical soliton perturbation with Kudryashov's generalized law of refractive index and generalized nonlocal laws by improved modified extended tanh method, Alexandria Engineering Journal. 61 (5) (2022) 3365–3374. (Impact factor: 3.732)
4. T. A. Nofal, I. Samir, N. Badra, A. Darwish, H. M. Ahmed, and A. H. Arnous, Constructing new solitary wave solutions to the strain wave model in micro-structured solids, Alexandria Engineering Journal. 61 (12) (2022) 11879–11888. (Impact factor: 3.732)

5. I. Samir, N. Badra, H. M. Ahmed, and A. H. Arnous, Solitary wave solutions for generalized Boiti–Leon–Manna–Pempinelli equation by using improved simple equation method, *International Journal of Applied and Computational Mathematics*. 2022. (Springer, Scopus: Q2)
6. I. Samir, N. Badra, A. R. Seadawy, H. M. Ahmed, and A. H. Arnous. Exact wave solutions of the fourth order non-linear partial differential equation of optical fiber pulses by using different methods, *Optik*. 230 (2021) 166313. (Impact factor: 2.443)
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10. I. Samir, N. Badra, H. M. Ahmed, and A. H. Arnous, Extracting Exact wave solutions for the coupled Fokas-Lenelss system via the improved modified extended tanh

scheme, (Submitted).

11. I. Samir, N. Badra, H. M. Ahmed, and A. H. Arnous, Extracting exact solutions for sixth-order dispersive non-linear Schrödinger's equation , (Submitted).

Contents

List of Publications	iii
Table of Contents	xi
List of Figures	xv
1 Preliminaries and basic concepts	1
1.1 Introduction	1
1.2 Traveling Wave Solutions	2
1.2.1 Solitary wave and solitons	3
1.2.2 Kink wave	4
1.2.3 Periodic wave solution	5
1.2.4 Peakons	6

1.2.5	Cuspons	7
1.3	Dispersion and Dissipation	7
1.4	Effective methods for solving NLPDEs	9
1.4.1	The modified (G'/G) expansion method	10
1.4.2	The modified Jacobi elliptic function method	13
1.4.3	The improved simplified equation method	14
1.4.4	The improved modified extended tanh method	17
1.4.5	Modified extended direct algebraic method	21
1.4.6	Modified extended mapping method	23
2	Exact wave solutions of the fourth order non-linear partial differential equation of optical fiber pulses by using different methods	27
2.1	Introduction	28
2.2	Exact solutions construction	29
2.2.1	On solving (2.5) using the modified Jacobi elliptic function method	30
2.2.2	On solving (2.5) using the modified (G'/G) expansion method . . .	33
2.3	Conclusion	37

3	Optical solitons and other solutions for coupled system of nonlinear Schrödinger's equation with parabolic nonlocal law of refractive index by using the improved modified extended tanh function method	38
3.1	Introduction	39
3.2	Governing model	41
3.3	Results	44
3.4	Graphic representation of solutions	52
3.5	Conclusion	55
4	Optical soliton perturbation with Kudryashov's generalized law of refractive index and generalized nonlocal laws by improved modified extended tanh method	56
4.1	Introduction	57
4.2	The exact wave solutions to the proposed model	58
4.3	Graphic representation of solutions	72
4.4	Conclusion	75
5	Computational extracting solutions for the perturbed Gerdjikov-Ivanov equation by using improved modified extended analytical approach	76
5.1	Introduction	77
