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The Role of Spins in Low Dimensional Structures

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Abstract

The study of the spin orbit interactions has gained a great attention in low dimensional structures, The three main types of spin orbit interactions are conventional, Dresselhaus and Rashba interactions. The conventional interactions result naturally due to the motion of the electron in its orbit. This motion, in turn, causes a coupling between the electron spin and its angular momentum, The Dresselhaus interactions arise in crystals which possess a lake of symmetry under reflection in a plane that involves at least one lattice point, The Rashba interactions are the most important type of spin orbit interactions that occur in low dimensional structures. They occur in two dimensional geometries. Their coupling coefficient is much higher than that of Dresselhaus interactions. They mainly depend on the inversion symmetry breaking in the direction perpendicular to the two dimensional structure.

The present work is mainly concerned with studying the Rashba spin orbit effect in different two dimensional geometries subject to a variety of confining potentials and external fields. The geometries which have been taken into consideration are a two dimensional discs subject to a parabolic confining potential and a perpendicular magnetic field. Also, an infinite quantum well wire subject to a finite confining potential and in the absence and presence of an axial magnetic field. Finally, the role of spin orbit interactions has been investigated in a spherical quantum dot with a radial parabolic confinement potential

and in the presence of external magnetic and electrical fields. The solution of the Schrödinger equation in the presence of the Rashba interactions has been derived by applying a new approach that differs from the one used in an earlier treatment. The wave function in the presence of these interactions has been expanded in terms of the eigenfunctions of the Hamiltonian in the absence of them.

The first problem considered is the case of a two dimensional disc subject to a parabolic confining potential and a perpendicular magnetic field. In this respect, we have criticized the work which has been performed by M. Kumar and his coworkers (Refs. [17], [24], [26]). It has been shown that an obvious contradiction exists between the form of the Rashba spin orbit Hamiltonian and the form of the energy eigenvalues and eigenfunctions they have been used. Also, we have amended the relations that have been reported in Kuan et al [21] between the coefficients involved in the expansions of the eigenfunctions in the presence of the Rashba interactions. We believe that the modified relations are more accurate than these reported in Kuan et al [21].

In the second problem, the effect of the Rashba spin orbit interaction has been investigated in an infinite cylindrical quantum well wire (QWW) with finite confining potential. The effect has been considered in the absence and presence of an axial magnetic field. The solution of the radial Schrödinger equation has been obtained in the wire and in the barrier, in the absence of the Rashba spin-orbit interactions. The general solution in the presence of these interactions has then been expanded in terms of the obtained two solutions and in terms of the step function. The study should, however, be performed in two dimensions. For this reason, the wave vector along the axis of the wire has been chosen to be zero. The orthogonality of the elements of the resulting basis has been proved. The dependence of the Rashba coupling coefficient on the electron effective mass has also been taken into consideration. The results have been applied to the case of $GaAs - Ga_{1-x}Al_xAs$ cylindrical quantum wire.

Finally, the role of the Rashba effect has been explored in a spherical quantum dot confined by a radial parabolic potential. Also, external parallel magnetic and electric fields have been applied. The solution of the Schrödinger equation in the presence of the Rashba interactions has been derived by applying an approach that differs from the one used in an earlier treatment (Refs. Vaseghi et al [53], [55]). The wave function in the presence of these interactions has been expanded in terms of the eigenfunctions of the Hamiltonian in their absence. In our opinion the new form introduced for the wave function presents the exact solution in a more accurate manner. The coefficients of expansion have been chosen either to depend on the three quantum numbers involved or on the principal quantum number only. The results have shown that the Rashba interactions have a considerable effect on the electron energy levels and on their splitting. The variation of this effect with the applied fields and the Rashba coupling strength has been investigated.

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INTRODUCTION

Chapter 1
INTRODUCTION

1.1 Review of the Role of Spin Orbit Interactions in Low Dimensional Structures

Nowadays, the spin orbit interactions have gained a great attention in low dimensional structures such as quantum wells (QWs), quantum well wires (QWWs) and quantum dots (QDs). The studies have covered both the theoretical and experimental points of view. The spin orbit interactions have a significant effect in such structures as they play an important role in spin transport and in spintronic devices [19, 29, 50, 51, 56, 58, 64]. The three main types of spin orbit interactions are conventional, Dresselhaus and Rashba interactions.

The conventional interactions result naturally due to the motion of the electron in its orbit. This motion, in turn, causes a coupling between the electron spin and its angular momentum (Griffiths [13] and Shankar [49]). Very few studies have been performed to explore such interactions in low dimensional structures. In Ref. [5, 38, 41, 62, 63] the conventional spin orbit interaction due to the presence of an impurity in a quantum dot was considered. Recently, Mikhail et al [33], [34] have investigated this type of interactions in single and multilayered quantum dots in the presence of central and off-central impurities.

The Dresselhaus interactions (Dresselhaus [9]) arise in crystals which possess a lake of symmetry under reflection in a plane that involves at least one lattice point. They have been studied by Lu and Li [30] in the presence of a magnetic field and by Vaseghi et al [54] in the presence of a spatial electric field.

The Rashba interactions (Rashba [43], [44]) are the most important type of spin orbit interactions that occur in low dimensional structures. They occur in two dimensional geometries. Their coupling coefficient is much higher than that of Dresselhaus interactions (Maiti et al [31]). They mainly depend on the inversion symmetry breaking in the direction perpendicular to the two dimensional structure. The symmetry breaking is represented by an electric field in the perpendicular direction which, in turn, generates a magnetic field. The required interaction results due to the coupling between the generated magnetic field and the electron spin.

The effect of Rashba spin-orbit interaction on the energy states splitting and on the electron spin transport has been investigated through a quantum one dimensional wire or waveguide in Refs. ([59, 60, 6, 18]). In these references, the authors have further considered the effect of the Rashba spin orbit interaction on the energy states splitting, excitation of spin waves and the spin transport of electrons. Also, the Rashba spin-orbit interaction has been studied in two dimensional discs by many authors (Ref. [57, 11, 61, 20]). Moreover, the phenomenon has been explored in a two dimensional electron gas in the presence of an external magnetic field. The magneto – quantum transport and magneto – thermodynamic properties of the system have been studied [12, 16, 14, 3, 21, 22]. The Rashba effect in the presence of both electric and magnetic fields has also been investigated in a one dimensional parabolic confinement QWW by Lahon et al [27] and Kumar et al [23], [25] and in a two dimensional electron system by Papp and Micu [39]. Moreover, the same problem was considered by Mehdiyev et al [32] on a circular cylinder of constant radius and by Vaseghi et al [53], [55] in a spherical quantum dot under a parabolic confinement potential.

In the last two decades considerable efforts have been devoted to explore the role of the Rashba spin-orbit interactions on the linear and nonlinear optical properties of two-dimensional geometries (Hassanabadi et al [15], Lahon et al [27], Kumar et al [24], [26], Jha et al [17], Nazari et al [36], Sakiroglu et al [47], Portacia et al [41], Vaseghi et al [53], [55]).

The combined effect of a hydrogenic impurity and Rashba spin-orbit interactions has further been considered by Vanitha et al [52] in a quantum well and by Safaei et al [45] in quantum nanowires. Moreover, the Rashba effect in the presence of an impurity and external magnetic field in a gaussian $GaAs$ quantum dot has been studied by Saini et al [46].

In spite of the fact that the Rashba coupling coefficient is much stronger than the corresponding Dresselhaus coupling coefficient both Rashba and Dresselhaus interactions have been studied together in Refs. [11, 20, 37, 39, 61].

The present work is mainly concerned with studying the Rashba spin orbit effect in different two dimensional geometries subject to a variety of confining potentials and external fields. The geometries which have been taken into consideration are a two dimensional disc subject to a parabolic confining potential and a perpendicular magnetic field. Also, an infinite quantum well wire subject to a finite confining potential and in the absence and presence of an axial magnetic field. Finally, the role of spin orbit interactions has been investigated in a spherical quantum dot with a radial parabolic confinement potential and in the presence of external magnetic and electrical fields.

Furthermore the dependence of the Rashba coupling coefficient on the electron effective mass has been included in a part of the application on the infinite QWW. This dependence has not been taken into consideration in any previous study. Two different values of the Rashba coupling coefficient in the wire and barrier have been used. These values have been taken to be inversely proportional to the values of the effective mass (Ref. [65]). A general form of the Rashba coupling coefficient has been introduced in terms of its two values and the step function. Such study is indeed important from a fundamental point of view regardless of the numerical changes that it leads to.

1.2 Outline of The Present Work

The thesis consists of four chapters and a list of references. The work is arranged in the following way:

In chapter 2 we have started by considering a two dimensional *GaAs* quantum dot (disc). The confining potential has been taken to be parabolic and an external magnetic field has been applied along the perpendicular direction to the disc. In section 2.1 we have mainly followed the work of Jha et al [17] and Kumar et al [24], [26]. However, in section 2.2 we have criticized their work concerning the Rashba effect where we have presented an obvious contradiction between the form of the Rashba spin orbit Hamiltonian and the form of the energy