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بسم الله الرحمن الرحيم

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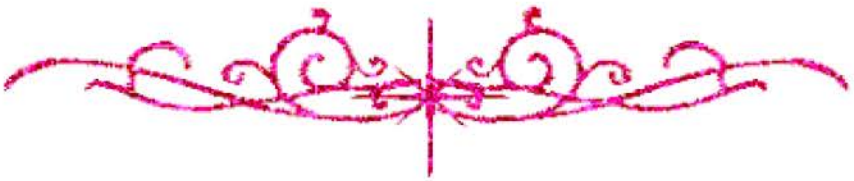
التوثيق الالكتروني والميكروفيلم



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قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها

علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات

يجب أن

تحفظ هذه الأقراص المدمجة بعيدا عن الغبار

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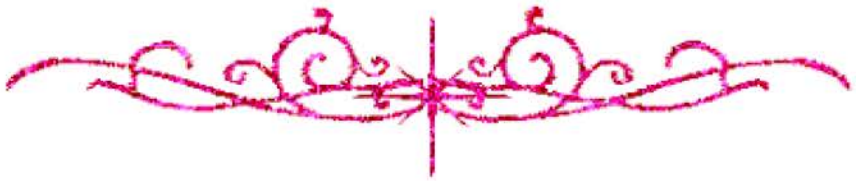
بعض الوثائق الأصلية تالفة



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بالرسالة صفحات

لم ترد بالأصل

HOSSAM MAGHRABY

Minufiya University
Faculty of Science
Department of Mathematics

A variational approach to asymptotic spectrum of multiparameter eigenvalue problems in Hilbert space

A Thesis

Submitted To Mathematics Department, Faculty of Science
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(Pure Mathematics)

By

Sayed Khalil Mohamed Marzok Elagan

Demonstrator
Mathematics Department , Faculty of Science
Minufiya University

Supervisors

Prof. Dr. Mohamed M. El-Sheikh
Math. Dept
Faculty of Science
Minufiya University
[ ]

Dr. Mahmoud H. Sallam
Math. Dept
Faculty of Scienc
MinufiyaUniversity
[ ]

لجنة الفحص والمناقشة:

[ ] ١- ا.د / محمد ابراهيم حسن أستاذ الرياضيات البحتة-كلية الهندسة- جامعة عين شمس

[ ] ٢- ا.د / محمود زكى رجب أستاذ الرياضيات البحتة-كلية العلوم- جامعة الزقازيق

[ ] ٣- ا.د / محمد محمود الشيخ أستاذ الرياضيات البحتة-كلية العلوم- جامعة المنوفية

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S.Elagan

Abstract

This thesis deals with the investigation of the asymptotic spectrum of a multiparameter problem in Hilbert space.Our discussion is based on estimates for eigenvalues derived from the minimum-maximum principle, this thesis includes three chapters.The first chapter entitled: A development of multiparameter spectral theory. In this chapter we presents a general introduction to multiparameter spectral theory, the definitions and the basic results which be needed in our study.The second chapter entiteld: Asymptotic spectrum of multiparameter eigenvalue problems, this chapter is concerned with the asymptotic spectrum of multiparameter eigenvalue problem under definiteness conditions.The third chapter entitled: The multiparameter Sturm-Liouville problem. In this chaper we shall consider as an ,application, a multiparameter Sturm-Liouville problem.

Contents

Chapter 1

ADEVELOPMENT OF MULTIPARAMETER SPECTRAL THEORY

1.1 Introduction

In recent years, much development has been taken place in the study of systems of equations in linear spaces involving more than one parameter . This study has become known as multiparameter spectral theory. Several authors have made important contributions to the theory.Here, we may cite the works of Atkinson [1], Browne [5], Binding [4], Sleeman [13], and Volkmer [16]. Many important results have been achieved. These results have applications, not only to system of ordinary differential equations, but also to a much wider range of linear systems ([11]).

However, multiparameter problems have appeared in the literature since the latter part of the 19th century. In fact, the multiparameter Sturm-Liouville problem in ordinary differential equations is nearly as old as the classical one parameter case . Therefore, it would seem worth

while to begin with the abstract approach to multiparameter problems which has been so fruitful in recent years.

1.2 The Abstract Approach to Multiparameter Problems

Let $H_r, r = 1, ..., k$ be separable Hilbert space with unit spheres

$$U_r = \{u_r \in H_r : \|u_r\| = 1\}, 1 \leq r \leq k.$$

Assume that

$$V_{rs} : H_r \to H_r, 1 \leq r, s \leq k$$

is self-adjoint bounded linear operator, and

$$T_r : H_r \supset D(T_r) \to H_r, r = 1, ..., k$$

is self adjoint operator, bounded below, and has compact resolvent.

The multiparameter problem which we consider, associated with the above array of operators is given by the system of equations

$$W_r(\lambda)x_r = 0, x_r \in D(W_r), r = 1, ..., k, \tag{1.1}$$

$$W_r(\lambda) : H_r \supset D(W_r) \to H_r,$$

where

$$W_r(\lambda) := T_r + \sum_{s=1}^{k} \lambda_s V_{rs}, \text{ and } \lambda = (\lambda_1, ..., \lambda_k) \in R^k$$