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شبكة المعلومات الجامعية التوثيق الالكتروني والميكروفيلم





# جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

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Ain Shams University

University College of Women

for Art, Science and Education

Mathematics Department

Duadic and Consta Skew Cyclic Codes Over Certain Frobenius Rings

THESIS

SUBMITTED FOR THE DEGREE

Of

DOCTOR OF PHELOSOPHY (PH.D.)

(PURE MATHEMATICS)

ВҮ

#### RAMEZ ZAHAR AL-SHORBASSI

Mathematics Department

University College of Women for

Art, Science and Education

Ain Shams University

#### **SUPERVISORS**

Prof. Dr. Safaa S. Beshy Prof. Dr. Gamal A. Fouad Ismail

Professor of Applied Mathematics Professor of Pure Mathematics

University College of Women for University College of Women for

Art, Science and Education Art, Science and Education

Prof. Dr. Mohammed M. Al-Ashker

Professor of Pure Mathematics

The Islamic University of Gaza

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Ain Shams University
University College of Women
for Art, Science and Education
Mathematics Department

# Ph.D. Thesis (PURE MATHEMATICS)

#### Title of thesis

### Duadic and Consta Skew Cyclic Codes Over Certain Frobenius Rings

#### Thesis supervisors

Prof. Dr. Safaa S. Beshy	Prof. Dr. Gamal A. Fouad Ismail
Professor of Applied Mathematics	Professor of Pure Mathematics
University College of Women for	University College of Women for
Art, Science and Education	Art, Science and Education

#### Prof. Dr. Mohammed M. Al-Ashker

Professor of Pure Mathematics The Islamic University of Gaza

## COURSES

The student has passed the following courses in partial fulfillment of requirement for Ph.D. degree:

- 1. Cryptography: Theory and practice  $(3^{rd} \text{ ed})$ , by Douglas R. Stinson. 2h per week
- 2. Elementary Number Theory:Primes, Congruences, and Secrets, by William Stein.

  2h per week

Head of Mathematics Department



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### ABSTRACT

RAMEZ Z. AL-SHORBASSI. Duadic and Consta Skew Cyclic Codes Over Certain Frobenius Rings. Doctor of Philosophy of Science dissertation (Pure Mathematics). University College of Women for Art, Science and Education, Ain Shams University.

This thesis focuses on study  $(\theta - \beta)$ -constacyclic codes and quadratic residue codes over the ring  $\mathcal{R} = F_p + uF_p + vF_p + uvF_p + v^2F_p + uv^2F_p$ .

In this thesis, we set some definitions and theorems which will be applied throughout the thesis, we study a class of rings called Frobenius rings. Linear codes, cyclic codes, Constacyclic codes, Skew cyclic codes, and Skew constacyclic codes are studied over the rings  $F_q + uF_q + vF_q + uvF_q$  and  $F_q + uF_q + vF_q + uvF_q + v^2F_q + uv^2F_q$ . Finally quadratic residue codes are studied over the rings  $F_p + uF_p + vF_p + uvF_p$ ,  $F_p + vF_p + v^2F_p + v^3F_p$  and  $F_p + uF_p + vF_p + uvF_p + v^2F_p + uv^2F_p$ .

**Keywords:** Linear Codes, Cyclic Codes, Constacyclic Codes, Skew Cyclic Codes, Skew Cyclic Codes, Skew Constacyclic Codes, Duadic Codes, Quadratic Residue Codes.