



شبكة المعلومات الجامعية  
التوثيق الإلكتروني والميكرو فيلم

# بسم الله الرحمن الرحيم



**HANAA ALY**



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التوثيق الإلكتروني والميكروفيلم



# شبكة المعلومات الجامعية التوثيق الإلكتروني والميكروفيلم



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# جامعة عين شمس

## التوثيق الإلكتروني والميكروفيلم

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**HANAA ALY**



Faculty of Education  
Department of Mathematics

# **Some problems of the peristaltic motion of nanofluids in the presence of different external effects**

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of Master Degree in Teacher Preparation in Science  
(Applied Mathematics)

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By

**Mohamed Maged Abd ElMoneim Eid**

B. Science & Education (Mathematics)  
Faculty of Education  
Ain Shams University (2015)

## **Supervised by**

**Prof. Dr. Nabil Tawfek El-dabe**  
Professor of Applied Mathematics  
Faculty of Education  
Ain Shams University

**Prof. Dr. Mohamed Yahya Abouzeid Nassar**  
Professor of Applied Mathematics  
Faculty of Education  
Ain Shams University

**Dr. Mona Abd El-hamed Ali Mohamed**  
Lecturer of Applied Mathematics  
Faculty of Education  
Ain Shams University

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Faculty of Education  
Department of Mathematics

**Candidate:** Mohamed Maged Abd ElMoneim Eid

**Thesis title:** Some problems of the peristaltic motion of nanofluids in the presence of different external effects.

**Degree:** Master Degree in Teacher Preparation in Science.  
(Applied Mathematics)

**Supervisors:**

| No. | Name  | Signature |
|-----|---|-----------|
| 1.  | <b>Prof. Dr. Nabil Tawfek El-dabe</b><br>Emeritus professor of Applied Mathematics<br>Department of Mathematics<br>Faculty of Education<br>Ain Shams University |           |
| 2.  | <b>Prof. Dr. Mohamed Yahya Abouzeid Nassar</b><br>Professor of Applied Mathematics<br>Department of Mathematics<br>Faculty of Education<br>Ain Shams University |           |
| 3.  | <b>Dr. Mona Abd El-hamed Ali Mohamed</b><br>Lecturer of Applied Mathematics<br>Department of Mathematics<br>Faculty of Education<br>Ain Shams University        |           |

# Abstract

This thesis concerned to study the peristaltic flow of nanofluid through a non-Darcy porous medium in a non-uniform inclined channel under the presence of some different external effects; that played an important role to control the flow. In addition to, it included the study of some different models of non-Newtonian fluid as power-law model (chapter 2), Bingham and Herschel Bulkley models (chapter 3 and 4). Also, the governing equations represent a system of nonlinear differential equations that were solved numerically by using Rung-Kutta-Merson method (chapter 2 and 3) and analytically by using homotopy perturbation method (chapter 4). Moreover, the effects of entering physical parameters on the obtained solutions were explained and discussed through a set of figures.

## List of publications

1. "MHD peristaltic flow of non-Newtonian power-law nanofluid through a non-Darcy porous medium inside a non-uniform inclined channel".

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3. "Peristaltic mixed convection slip flow of a Bingham nanofluid through a non-Darcy porous medium in an inclined non-uniform duct with viscous dissipation and radiation".

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# Summary

# Summary

The study of non-Newtonian nanofluids flow in different geometric ducts gained great attention, due to their numerous biological, chemical and industrial applications. Some of these applications are discussed in the introduction. Moreover, the study of nanofluids flow under external effects is a very important topic as these external effects can control and enhance the velocity of the flow as well as its temperature and concentration.

In this thesis, we are concerned to study the peristaltic flow of incompressible non-Newtonian nanofluid through a uniform ducts in the presence of non-Darcy porous medium. It is also noticed that the flow is unsteady in the laboratory frame (fixed frame), but we separated the time by changing the flow to the moving frame with wave speed. Moreover, the flow is affected by numerous external forces, such as: uniform external magnetic field, thermal radiation, chemical reaction, viscous dissipation, Ohmic dissipation and other external forces. Thereby the governing equations for velocity, temperature and nanoparticles concentration represent a system of partial differential equations that are very difficult to be solved by ordinary methods. Therefore, In chapters 2 and 3 the governing equations are solved numerically by using Runge-Kutta-Merson method under the help of Mathematica program, version 12.0.0, while in chapter 4 the governing equations are solved analytically by using homotopy perturbation method (HPM).

The thesis consists of four chapters as follows:

## Chapter 1

This chapter represents a general introduction to cover the most important and fundamental items in the thesis:

- Introduction to non-Newtonian fluids and some of their models.
- Introduction to peristaltic flow and some of its application.

- Introduction to nanofluids and some of their application.
- Introduction to Magnetohydrodynamics flows and some of their application.
- Introduction to fluid flow through a non-Darcy porous medium and some of its applications.

## Chapter 2

The aim of this chapter is to study the peristaltic transport of incompressible non-Newtonian nanofluid flow through a non-uniform inclined channel. The fluid in this chapter obeys power-law model in the presence of non-Darcy porous medium. Moreover, the effects of thermal radiation, heat generation, Ohmic dissipation and a uniform external magnetic field are taken in consideration. On the other hand, the governing equations that describe the velocity, temperature and nanoparticles concentration are simplified under the assumptions of long wave length and low-Reynolds number. These equations are solved numerically by using Rung-Kutta-Merson method with the help of shooting and matching technique. It is also noticed that the solutions are obtained as functions of the physical parameters entering the problem. Thereby, these parameters play an important role to control and enhance the obtained solutions. Finally, we discussed the effects of these parameters and illustrated them through set of figures.

**It is worth to note that the contents of this chapter are published in a scientific journal "Archive of applied mechanics" with impact factor 1.547**

## Chapter 3

The aim of this chapter is to study the peristaltic transport of incompressible non-Newtonian nanofluid flow through a non-uniform vertical channel. The fluid in this chapter obeys Herschel Bulkley model in the presence of non-Darcy porous medium. Moreover, the effects of thermal radiation, heat generation, Ohmic dissipation and a uniform