



Ain Shams University  
Faculty of girls for  
Arts, Science and Education  
Mathematics Department

# GEOMETRIC SOLUTIONS OF PHYSICAL PROBLEMS

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BY

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### **Geometric Solutions of Physical Problem**

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TO MY LOVELY BOSBOS





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# Abstract

In the present Thesis, we have chosen a phenomena, known in the literature as the Aharonov-Bohm effect, to be treated geometrically. The problem is that this phenomena is widely known as a pure quantum effect. A new equations of motion of a charged test particle is derived. The method used to derive this equation is known in the literature as Bazanski-method. This method and the conventional method are known in the literature to give identical results in Riemannian geometry. In the present thesis, we have used the Bazanski method, but in Riemannian geometry modified by a vector field. The thesis shows that, in this case we get a new path equation, which can account for the Aharonov-Bohm effect, geometrically.



# Summary

The current thesis contains Mainly three Chapters:

## **Chapter I: The Aharonov-Bohm Effect.**

This Chapter reviews briefly historical background of the Aharonov-Bohm, A-B, effect. We give the first steps of theoretical predictions of this effect, qualitatively. Also, we give a brief account on Aharonov and Bohm contribution of interpreting the new phenomenon qualitative and quantitative. The experimental evidence of the phenomenon is mentioned here. These experiments confirm this phenomenon qualitative and quantitative too. Discussion, and the aim of the present work are given at the end of this Chapter.

## **Chapter II: The Mathematical Treatments**

This Chapter shows the difference between the two main points of view which dealt with the A-B effect, in the first view, we briefly reviewed the explanation of Maxwell theory of the effect which gives a zero result. In the second view, we explain the studied phenomenon using Schrödinger equation, which gives a direct evidence with the experimental result. We also clarify the contradiction between the above two methods, furthermore, we show the main reason of why the phenomenon dealt as a pure quantum effect. Discussion and criticism are given at the end of this Chapter.