



شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم

بسم الله الرحمن الرحيم



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شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكرو فيلم



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التوثيق الإلكتروني والميكروفيلم

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Phase Space Analysis of Friedmann–Lemaitre–Robertson–Walker Cosmology

A Thesis submitted in partial fulfillment of the requirement for
the degree of

Master of Science in Physics

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