



شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم

بسم الله الرحمن الرحيم



MONA MAGHRABY



شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم



شبكة المعلومات الجامعية التوثيق الإلكتروني والميكروفيلم



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شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم

جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

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AIN SHAMS UNIVERSITY
FACULTY OF ENGINEERING
Structural Engineering

Development of a New Finite Element Solver Using a Guided Relaxation Technique

A Thesis submitted in partial fulfillment of the requirements of the degree
of Master of Science in Civil Engineering

(Structural Engineering)

By

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B.Sc. in Civil Engineering (Structural Engineering)

Faculty of Engineering, Ain Shams University, June 2016

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Statement

This thesis is submitted as a partial fulfillment of Master of Science in Civil Engineering, Faculty of Engineering, Ain Shams University.

The author carried out the work included in this thesis, and no part of it has been submitted for a degree or a qualification at any other scientific entity.

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Thesis Summary

Among the numerical methods for solving engineering problems, is the finite element method (FEM) whose applications in various fields of science and engineering are continually growing resulting in a similar increase in the size and sophistication of the models. Consequently, there is a mounting need for faster and more efficient finite element analysis (FEA) algorithms since reaching the solution to the system of linear equations in FEM is usually the most exhausting and computationally demanding part through the process. There are two main classes of algorithms which are Direct and Iterative solvers. The performance of any technique changes depending on the scale of the system to be analyzed and the properties of the coefficient matrix, which in turn affect the selection between the solvers depending on some standards, i.e., the solver's performance, such as the results' accuracy, the required storage, amount of computation and performance speed. Unfortunately, these requirements are usually contradictory, hence it is still essential to find out new effective solutions as there is no single technique that outperforms the others in all cases. Generally, the direct methods tend to demand a remarkably large memory space and a great amount of calculation for huge problems, so they take long run time. Thus, an iterative solver, that requires relatively less memory space, is more desirable in these cases. Moreover, iterative solvers are generally simpler to program. After considering a number of iterative analysis techniques, the classical relaxation method was selected as a starting point for this research. The main objective of this research is to develop a new finite element solver using a Guided Relaxation (GR) technique, which is an iterative method based on standard force-relaxation methods in addition to being guided by certain innovative Relaxation Modes that were carefully chosen to boost the rate of convergence. Furthermore, some procedure modifications are developed to improve the performance of GR. The idea and algorithm of GR as well as the development sequence of Relaxation Modes and procedure modifications are elaborated in both 2D and 3D analyses along with illustrating the impact of each Relaxation Mode and procedure modification on GR's performance. After software implementation of GR's algorithm in addition to some other classical and modern iterative techniques, the rate of convergence of GR is compared to that of those techniques for assessment in different 2D cases. Then, a large parametric study is conducted

and provided to assess GR by comparing it with Conjugate Gradient (CG) method in 2D and 3D analyses depending on various parameters and conditions. Finally, conclusions and recommendations concerning the suitability of usage of both GR and CG techniques are provided.

Key Words: finite element method, direct and indirect solvers, classical and modern iterative methods, classical relaxation, Conjugate Gradient.

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