



شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم

بسم الله الرحمن الرحيم



MONA MAGHRABY



شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم



شبكة المعلومات الجامعية التوثيق الإلكتروني والميكروفيلم



MONA MAGHRABY



شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم

جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها
علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات



يجب أن

تحفظ هذه الأقراص المدمجة بعيدا عن الغبار



MONA MAGHRABY



Cairo University

MULTILEVEL MONTE CARLO METHODS FOR SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

By

Eng. Shady Ahmed Nagy

A thesis submitted to the
Faculty of Engineering at Cairo University
In partial Fullfilment of the
Requirements for the degree of
Master of science
In
Engineering Mathematics

FACULTY OF ENGINEERING, CAIRO UNIVERSTY
GIZA, EGYPT
2020

MULTILEVEL MONTE CARLO METHODS FOR SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

By

Eng. Shady Ahmed Nagy

A thesis submitted to the
Faculty of Engineering at Cairo University

In partial Fullfilment of the
Requirements for the degree of

Master of science

In

Engineering Mathematics

Under the supervision of

Prof. Dr. Mohamed A. El-Beltagy

Department of Mathematics
and Engineering Physics.,
Faculty of Engineering,

Cairo University

Dr. Mohamed Wafa

Department of Mathematics
and Engineering Physics.,
Faculty of Engineering,

Cairo University

FACULTY OF ENGINEERING, CAIRO UNIVERSTY
GIZA, EGYPT
2020

MULTILEVEL MONTE CARLO METHODS FOR SOLUTION OF STOCHASTIC DIFFERENTIAL EQUATIONS AND ITS APPLICATIONS

By

Eng. Shady Ahmed Nagy

A thesis submitted to the
Faculty of Engineering at Cairo University
In partial Fullfilment of the
Requirements for the degree of
Master of science
In
Engineering Mathematics

Approved by the Examining Committee:

Prof. Dr. Mohammed A. El-Beltagy, Thesis Main Advisor

Prof. Dr. Maha Amin Hasanin, Internal Examiner

Prof. Dr. Nasser Hassan Sweilam, External Examiner
Faculty of Science, Cairo University

FACULTY OF ENGINEERING, CAIRO UNIVERSTY
GIZA, EGYPT
2020

Engineer's Name: Shady Ahmed Nagy
Date of Birth: 22/12/1987
Nationality: Egyptian
E-mail: Shady.nagy@guc.edu.eg
Phone: 01007690927
Address: AL Khalifa, Cairo, 1212
Registration Date: 01/10/2013
Awarding Date:10/2020
Degree: (Master of Science)
Department: Engineering Mathematics
Supervisors:



Prof. **Mohammed A. El-Beltagy**
Dr. **Mohammed Wafa**

Examiners:

Prof. **Mohammed A. El-Beltagy** (Thesis main advisor)
Prof. **Maha Amin Hasanin** (Internal examiner)
Prof. **Nasser Hassan Sweilam** (External examiner)
(Faculty of Science, Cairo University)

Title of Thesis:

Multilevel Monte Carlo methods for solution of stochastic differential equation and its applications.

Key Words:

Stochastic differential equations; Stochastic processes; Numerical analysis; Multilevel Monte Carlo; Low discrepancy sequences.

Summary:

Monte Carlo simulation is wide using in solving stochastic differential equations. Stochastic random samples represent by different random points in Monte Carlo random generation. The development of Multilevel Monte Carlo (MLMC) introduced by Giles to simulate different stochastic differential equations on different time grids by low cost and high convergence rate, also it minimizes the variance. We simulate and compare different type of stochastic differential equations on MLMC depending on Quasi-Monte Carlo of Halton sequence. We apply MLMC in different types of ordinary SDEs as additive and multiplicative one to enhance cost by changing the random sample to be generated by different quasi-random numbers. Also, we use a different type of quasi-random numbers by Component by Component (CBC) algorithm that generates different random numbers by the concept of lattice rule. When we apply it in stochastic Burgers' equations, the instability appears in simulation by doesn't achieve the decreasing in cost despite the minimum time of CBC numbers that generate the stochastic samples.

Disclaimer

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute. I further declare that I have appropriately acknowledged all sources used and have cited them in the references section.

Name: Shady Ahmed Nagy

Date:

Signature:

Acknowledgment

I would first like to thank my thesis advisor Professor Mohammed EL-Beltagy of the Engineering Mathematics at faculty of engineering Cairo University. The door to Prof. EL-Beltagy office was always open whenever I ran into a trouble spot or had a question about my research or writing. He consistently allowed this paper to be my own work, but steered me in the right the direction whenever he thought I needed it. Finally, I must express my very profound gratitude to my Mother for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them. Thank you.

TABLE OF CONTENTS

LIST OF FIGURES.....	VI
LIST OF TABLES.....	VII
LIST OF PUBLICATIONS.....	IX
ABSTRACT	X
CHAPTER 1: Introduction	1
CHAPTER 2: Stochastic processes	3
2.1. Continuity of stochastic processes	4
2.2. First and second moment of stochastic processes	5
2.3. Martingales	5
2.4. Markov property	7
2.5. Brownian motion.....	7
2.6. Ito integral	9
2.7. Stochastic differentials equations.....	12
2.8. Examples of SDEs.....	13
2.9. Grownwall's inequality.....	16
CHAPTER 3: Numerical solutions of stochastic differential equations	17
3.1. Strong convergence.....	18
3.2. Weak convergence.....	18
3.3. Euler numerical scheme	18
3.3.1. Euler Maruyama method for geometric stochastic differential equation.....	19
3.3.2. Euler Maruyama method for Langevin equation.....	20
3.3.2. Euler Maruyama method for Verhulst equation.....	21
3.4. Milstein numerical scheme	22
3.4.1. Milstein approximation for geometric stochastic differential equation.....	23

3.4.2. Milstein approximation for Langvein equation.....	24
3.4.2. Milstein approximation for Verhulst equation.....	25
3.5. Stochastic Taylor expansion.....	25
3.5.1. Order 1.5 strong Taylor expansion.....	26
3.5.2. Weak Taylor expansion	26
3.6. Strong order 1.0 Runge Kutta method.....	26
3.6.1. Weak order 2.0 Runge Kutta method.....	27
CHAPTER 4: Monte Carlo simulations.....	28
4.1. Variance reduction methods	28
4.1.1. Control variates.....	28
4.2. Monte Carlo Method	29
4.3. Latin hypercube sampling	31
4.4. Quasi Monte Carlo Method	33
4.4.1. Lattice rules	34
4.4.2. Low discrepancy sequences.....	34
4.4.2.1. Halton sequences.....	35
CHAPTER 5: Multilevel Monte Carlo (MLMC).....	37
5.1. MLMC Theorem	38
5.2. Multilevel algorithm	40
5.3. Improved Multilevel Monte Carlo.....	43
5.4. Multilevel Quasi Monte Carlo.....	43
5.5. Quantity of interest	44
5.5.1. Geometric Brownian motion	46
5.5.2. Ornstein Uhlenbleck equation	47
5.5.3. Vasciek Model	49
5.5.4. Verhulst equation.....	51
5.5.5. Cox Ingersoll Ross model.....	53
5.5.6. Allen-Cahn equation	55

CHAPTER 6: Simulations and Results	57
6.1. MLMC on different SDEs	57
6.2. MLMC on nonlinear PDEs (Stochastic Burgers).....	66
6.2.1. Component by Component algorithm (CBC)	68
6.2.2. Simulations	70
6.3. MLMC simulations on particle filters and extended Kalman filters	75
6.3.1. Particle filters	75
6.3.2. Extended Kalman filters	76
6.3.3. Simulations	77
CHAPTER 7: Summary and future work	81
References	82
Appendix	86

List of Figures

Fig (1) : Exact and Euler approximated of geometric brownian motion stochastic differential equation.	20
Fig (2) : Euler-Maruyama discretization of Langevin equation.	21
Fig (3) : Exact and Euler-Maruyama discretization of Verhulst equation.	21
Fig (4) : 1000 samples of Verhaulst equation.	22
Fig (5) : Milstein approximation of geometric brownian motion.	23
Fig (6) : Milstein approximation of Langevin equation.	24
Fig (7) : Exact and Milstein approximation of Verhaulst equation.	25
Fig (8) : 100 points of latin hypercube sequence.	32
Fig (9) : 1000 Halton sequence points.	36
Fig (10) : Optimal values of M for MLMC.	42
Fig (11) : MLMC for Geometric brownian motion.	46
Fig (12) : MLMC for Ornstein Uhlenbleck equation QOI all output vector.	47
Fig (13) : MLMC for Ornstein Uhlenbleck equation QOI summation output vector. ..	48
Fig (14) : MLMC for Vasciek model QOI all output vector.	49
Fig (15) : MLMC for Vasciek model QOI summation output vector.	50
Fig (16) : MLMC for Verhaulst equation QOI all output vector.	51
Fig (17) : MLMC for Verhaulst equation QOI summation output vector.	52
Fig (18) : MLMC for Cox ingresol Ross model QOI all output vector.	53
Fig (19) : MLMC for Cox ingresol Ross model QOI summation output vector.	54
Fig (20) : MLMC for Allen Cahn equation QOI integral of u	55
Fig (21) : MLMC for Allen Cahn equation QOI integral of u^2	56
Fig (22) : 1000 halton points on the left, normally distributed in the middle, pseudo normally distributed on the right.	59
Fig (23) : MLMC cost with pseudo random generator and based Halton points for Vasciek model.	23

Fig (24) : MLMC cost with pseudo random generator and based Halton points for Inhomogenous SDEs.....	62
Fig (25) : MLMC cost with pseudo random generator and based Halton points for Variable coefficient SDEs.....	63
Fig (26) : MLMC cost with pseudo random generator and based Halton points for Verhaultst SDEs.....	65
Fig (27) : MLMC using CBC algorithm on 1D stochastic Burger with $\sigma = 0.1, N = 10^5$	71
Fig (28) : MLMC using CBC algorithm on 1D stochastic Burger with $\sigma = 0.5, N=10^4$	72
Fig (29) : MLMC using Pseudo random generator on 1D Stochastic Burger equation with $\sigma = 0.1, N=10^5$	73
Fig (30) : MLMC using Pseudo random generator on 1D Stochastic Burger equation with $\sigma = 0.5, N=10^5$	74
Fig (31) : EKF-MLMC simulation using noise intensity $\sigma = 0.5, N = 10^5$	78
Fig (32): Particle-filter-MLMC simulation using noise intensity $\sigma = 0.5, N = 10^5$	80

List of Tables

Table (1) : Cost of MLMC and Std MC based on pseudo random genrator for Vasciek Interest model.....	60
Table (2) : Cost of MLMC and Std MC based on Halton points for Vasciek Interest model.....	60
Table (3) : Cost of MLMC and Std MC based on pseudo random genrator for Inhomogenous SDEs.....	61
Table (4) : Cost of MLMC and Std MC based on Halton points for Inhomogenous SDEs.	62
Table (5) : Cost of MLMC and Std MC based on for pseudo random genrator Variable coefficents SDEs.....	63
Table (6) : Cost of MLMC and Std MC based on Halton points for Variable coefficents SDEs.	64
Table(7) : Cost of MLMC and Std MC based for pseudo random genrator Verhulst SDEs.	65
Table(8) : Cost of MLMC and Std MC based on Halton points for Verhulst SDEs.	65
Table(9) : Cost of MLMC and Std MC using CBC algorithm on 1D stochastic Burger with $\sigma = 0.1, N = 10^5$	72
Table (10) : Cost of MLMC and Std MC using CBC algorithm on 1D stochastic Burger with $\sigma = 0.5, N = 10^4$	73
Table (11) : Cost of MLMC and Std MC Pseudo random generator on 1D stochastic Burger with $\sigma = 0.5, N = 10^4$	74
Table (12) : Cost of MLMC and Std MC using Pseudo random generator on 1D Stochastic Burger equation with $\sigma = 0.5, N=10^5$	75
Table (13): Cost of simulation MLMC and Std MC using noise intensity $\sigma = 0.5, N = 10^5$	79
Table (14): Cost of Particle filters MLMC and MC-Std cost using noise intensity $\sigma = 0.5, N = 10^5$	79