بسم اللّه الرحمن الرحيم

شبكة المعلومات الجامعية

التوثيق الالكتروني والميكروفيلم

جامعة عين شمس

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قسم

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Cairo University

A CRITICAL STUDY OF HIGHER ORDER DISCONTINUOUS FINITE ELEMENT METHODS FOR SOLUTION OF EULER EQUATIONS

By

Yasien Essameldin Saadeldin Abdelaziz Ali Shaaban

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Engineering Mathematics

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
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Title of Thesis:

A CRITICAL STUDY OF HIGHER ORDER DISCONTINUOUS FINITE ELEMENT METHODS FOR SOLUTION OF EULER EQUATIONS

Key Words:
higher order discontinuous finite element methods for unstructured grids; Euler equations; von Neumann stability analysis; polynomial-based approximation; spatial discretization.

Summary:
This thesis presents a critical study for higher order discontinuous finite element methods. This study includes flux reconstruction approach, which includes discontinuous Galerkin method and spectral difference method. The study is conducted in the light of Von Neumann stability analysis. Hence, two-dimensional solver for quadrilateral grid has been developed. Then, a criticism of the aforementioned method is presented based on Von Neumann analysis. This criticism shows that the utilization of polynomial based approximation does not always yield the well-established order of accuracy in literature. Also, it shows that Euler model is second order accurate as a consequence of modelling error. Hence, the utilization of higher order accurate numerical methods does not make sense in solving the Euler equations. Finally, a new development for finite difference method is proposed. This development enables us to get a second order accurate solution without seeking numerical boundary conditions.

Disclaimer

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute.

I further declare that I have appropriately acknowledged all sources used and have cited them in the references section.

Name: Yasien Essameldin Saadeldin Abdelaziz Date: 12/1/2021

Signature: Yasien Abdelaziz

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Dedication

This work is dedicated to all professors who taught me throughout undergraduate and postgraduate levels. Thank you indeed, dear professors!

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Nomenclature

v	the conserved scalar quantity.
g	the flux in x direction.
x	the spatial variable.
t	time.
L	the domain.
L_k	the k^{th} element.
$v_k^{\delta D}$	the approximate solution over element L_k.
g_k^{δ}	the approximate flux function over element L_k.
L_S	standard element.
ξ	the spatial variable in standard element.
x_k	the initial point of finite element k.
x_{k+1}	the terminal point of finite element k.
$\Theta_k(\xi)$	linear mapping from standard element to physical element L_k.
$\hat{v}^{\delta D}$	the transformed approximate solution.
$\hat{g}^{\delta}$	the transformed approximate flux function.
J_k	the Jacobian of transformation associated to physical element L_k.
l_j	Lagrange polynomials
$\hat{v}_i^{\delta D}$	the value of the transformed approximate solution at solution point ξ_i within the standard element.
$\hat{g}_i^{\delta D}$	the value of the transformed approximate flux function at solution point ξ_i within the standard element.
$\hat{g}^{\delta D}$	the transformed approximate discontinuous flux function.
$v^{\delta D}$	the approximate physical solution found by transforming transformed approximate solution back to physical domain.
$v_L^{\delta D}$	the approximate physical solution at left end point of the element.
$v_R^{\delta D}$	the approximate physical solution at right end point of the element.
$v_{e,-}^{\delta D}$	the left state value at interface e.
${v_{k,R}}^{\delta D}$	the left state value of element L_k.
$v_{e,+}^{\delta D}$	the right state value at interface e.
${v_{k+1,L}}^{\delta D}$	the left state value of element L_k.
$\hat{g}_L^{\delta I}$	the transformed interface numerical flux at left end of the standard element.
$\hat{g}_L^{\delta D}$	the transformed discontinuous flux at left end of the standard element.
$\hat{g}_R^{\delta I}$	the transformed interface numerical flux at right end of the standard element.
$\hat{g}^{\delta C}$	the transformed corrected flux function.
P_L	the left flux correction function.
P_R	the right flux correction function.
h_n	the Legendre polynomial of degree n.
c	the parameter defining the correction function.
n	the degree of the utilized polynomial.
k	wave number.
h	the element length.
$\boldsymbol{N}$	differentiation matrix composed by Lagrange polynomial bases.
$\mathbf{P}_{\xi L}$	vector containing values of the derivative of the left correction function at solution points.