

شبكة المعلومات الجامعية التوثيق الإلكتروني والميكروفيلو

# بسم الله الرحمن الرحيم





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شبكة المعلومات الجامعية التوثيق الإلكتروني والميكروفيلو



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# جامعة عين شمس التوثيق الإلكتروني والميكروفيلم قسم

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تحفظ هذه الأقراص المدمجة بعيدا عن الغبار



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## Study of Some Possible Textures of Neutrino Mass Matrix

A Thesis submitted in partial fulfillment of the requirement for the degree of

## **Master of Science in Physics**

Ву

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To my beloved son..

#### Abstract

Neutrinos are elementary particles that have spin  $\frac{1}{2}$  integer, so they are called fermions. They have small masses compared to the other leptons  $(e, \mu, \tau)$ . Even though neutrinos are one of the most abundant particles in the universe, they are incredibly difficult to detect. They have neither an electric charge nor a color charge. Thus, they interact with matter only via weak and gravitational interactions. Neutrinos are created by different sources such as in beta decay of the nucleus, in nuclear reactions that occur in the core of the stars, in nuclear reactions inside the core of the reactors, during the explosion of the stars, etc. The first one who postulated the existence of neutrinos was Pauli in 1930 to preserve energy, momentum, and angular momentum conservation in beta decay. In 1956, neutrinos were first detected by Frederick Renines, F.B.Harrison, H.W.Kruse, and A.D.McGuire [1]. There are three different types of neutrinos which are called flavors  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  produced along with e,  $\mu$  and  $\tau$  in the weak interactions. For many years physicists considered a neutrino to be a massless particle, but after neutrino oscillations observations [2, 3, 4, 5, 6], we have to consider neutrinos as massive particles, and their mass eigenstates are not degenerate. The interactions between neutrinos and other elementary particles are described within the standard model of the elementary particle physics, which is  $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$  gauge theory. However, the matter content of the standard model does not contain right-handed neutrinos. Therefore, the neutrinos are massless particles in that model, but this assumption contradicts the neutrino oscillation observations. To overcome this problem, we have to go beyond the standard model to explain naturally how neutrinos acquire small masses compared to the other leptons. Therefore, the neutrino oscillations were the first firm sign of physics beyond the standard model.

The neutrino oscillation experiments give clear and strong evidence that the neutrinos are massive and lepton flavors are mixed. If we assume that neutrino is a Majorana type and also we are working on the basis, where the charged lepton mass matrix is diagonal, all mixing comes solely from the neutrino sector. The neutrino mass matrix is in general a complex symmetric matrix that has 12 real parameters. It can be parametrized by three mixing angles, three real masses, and six complex phases. One can absorb three complex phases out of the six one by rephasing both left and right-handed charged leptons. The experiments put constraints on the mixing angles  $(\theta_{12}, \theta_{23}, \theta_{13})$ , three neutrino masses  $(m_1, m_2, m_3)$ , and the Dirac phase  $\delta$ . However, there are no experimental constraints on the Majorana phases  $\rho$  and  $\sigma$  till now.

There exist many phenomenological models have been presented to reduce the number of free parameters such as zero textures [7, 8, 9, 10], zero minors [11, 12], vanishing traces [13, 14], equality textures [15, 16], hybrid textures [17, 18, 19],  $\mu - \tau$  symmetry textures [20, 21], etc.

The plan of the thesis is as follows: In chapter 1, we present the standard model of elementary particles as a gauge theory and concentrate on the  $SU(2)_L \otimes U(1)_Y$  gauge sector. We also discuss how the elementary particles acquire their mass without spoiling the gauge symmetry via the Higgs mechanism. In the second chapter, we explain the theory of the neutrino oscillations together with type-I and type-II seesaw mechanisms to show how observed neutrinos acquire small masses. In the last chapter, we introduce a phenomenological and analytical study of the one vanishing subtrace model. We find that all six possible textures can accommodate the experimental data. We also find that four singular textures are viable in the case of inverted hierarchy. Finally, we introduce different symmetry realization methods to enforce a vanishing subtrace condition in the neutrino mass matrix. The realization methods used in our study are called direct and indirect. In the direct method, we use  $Z_2 \times Z_6$ ,  $Z_2 \times Z_2'$ , and  $Z_2 \times Z_4 \times U(1)^3$  symmetries within type-I, type-II, and type-I+II respectively in order to realize four viable textures. In the indirect method, we obtain the desired symmetry assignments for the matter fields that impose the texture condition from the counterparts in the one zero texture model. We use  $Z_8 \times Z_2$  and  $Z_5$  symmetries within type-I and type-II seesaw scenarios. However,  $Z_{12} \times Z_2$  is used to realize some viable singular textures.

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# Contents

| Li | List of Tables |  |            |
|----|----------------|--|------------|
| Li | st of          | Figures  | 6          |
| 1  | Sta            | ndard model of elementary particle physics                               | 9          |
|    | 1.1            | Introduction   | Ö          |
|    | 1.2            | Leptons and currents in $SU(2)_L \otimes U(1)_Y$ model                   | 10         |
|    | 1.3            | The gauge invariant lagrangian in the electroweak theory                 | 12         |
|    | 1.4            | The Higgs mechanism  | 13         |
|    | 1.5            | Extension to more than one lepton family                                 | 23         |
|    | 1.6            | Extension to quarks  | 24         |
|    | 1.7            | Problems in the standard model   | 31         |
| 2  | Neı            | atrino masses and neutrino oscillation                                   | 33         |
|    | 2.1            | Introduction   | 33         |
|    |                | 2.1.1 Solar neutrinos  | 35         |
|    |                | 2.1.2 The super–Kamiokande experiment                                    | 36         |
|    |                | 2.1.3 The SNO experiment   | 37         |
|    | 2.2            | Neutrino oscillations of the two flavors                                 | 38         |
|    | 2.3            | Neutrino oscillation of the three flavors                                | 42         |
|    | 2.4            | CP and T symmetries violation in the neutrino oscillations               | 45         |
|    | 2.5            | Mechanisms of the neutrino mass  | 46         |
|    | 2.6            | Seesaw mechanism   | 47         |
|    | 2.7            | The lepton mixing matrix   | 49         |
| 3  | Tex            | ture of single vanishing subtrace in the neutrino mass matrix            | <b>5</b> 3 |
|    | 3.1            | Introduction   | 53         |
|    | 3.2            | Texture of single vanishing subtrace                                     | 54         |
|    | 3.3            | Numerical analysis for nonsingular textures                              | 55         |
|    |                | 3.3.1 Texture $\mathbf{C}_{11}$ : Vanishing of $M_{\nu 22} + M_{\nu 33}$ | 58         |
|    |                | 3.3.2 Texture $C_{12}$ : Vanishing of $M_{0.21} + M_{0.22}$              | 61         |

| Bi | Bibliography |   |     |
|----|--------------|---|-----|
| 4  | Sun          | nmary and Conclusion 1  | 13  |
|    | 3.9          | Indirect realization of type II seesaw with $Z_5$ symmetry                                | 100 |
|    |              | 3.8.1 Indirect realization for the singular textures with $Z_{12} \times Z_2$ symmetry    |     |
|    | 3.8          | Indirect realization of type-I seesaw with $Z_8 \times Z_2$ symmetry                      | 90  |
|    | 3.7          | Direct realization by using Type I+II seesaw with $Z_2 \times Z_4 \times U(1)^3$ symmetry | 86  |
|    | 3.6          | Direct realization by using type-II seesaw with $Z_2 \times Z_2'$ -symmetry               | 82  |
|    | 3.5          | Direct realization by using type-I seesaw with $Z_2 \times Z_6$ -symmetry                 | 78  |
|    | 3.4          | Numerical analysis for singular textures  | 71  |
|    |              | 3.3.6 Texture $C_{33}$ : Vanishing of $M_{\nu 11} + M_{\nu 22}$                           | 69  |
|    |              | 3.3.5 Texture $C_{23}$ : Vanishing of $M_{\nu 11} + M_{\nu 23}$                           | 67  |
|    |              | 3.3.4 Texture $\mathbf{C}_{22}$ : Vanishing of $M_{\nu 11} + M_{\nu 33}$                  | 65  |
|    |              | 3.3.3 Texture $C_{13}$ : Vanishing of $M_{\nu 21} + M_{\nu 23}$                           | 63  |

# List of Tables

| 1.1 | Lepton content in the electroweak theory  | 12 |
|-----|---|----|
| 1.2 | quark content in electroweak theory   | 25 |
| 2.1 | The experimental bounds for the oscillation parameters at 1-2-3 $\sigma$ -levels, taken from the global |    |
|     | fit to neutrino oscillation data [22]. Normal and Inverted Hierarchies are respectively denoted         |    |
|     | by NH and IH  | 52 |
| 3.1 | The various prediction for the patterns of one vanishing subtrace textures des-                         |    |
|     | ignated by $C_{11}, C_{12}, C_{13}, C_{22}, C_{23}$ and $C_{33}$  | 57 |
| 3.2 | The various predictions for the patterns of one vanishing subtrace textures and                         |    |
|     | vanishing $m_3$ designated by $C_{12}, C_{13}, C_{22}$ and $C_{33}$                                     | 73 |
| 3.3 | The $Z_2 \times Z_6$ symmetry realization for the $\mathbf{C}_{33}$ pattern within type-I seesaw        |    |
|     | scenario. $\Phi$ are five Higgs doublets, $D_L$ refers to the flavor three left handed                  |    |
|     | lepton doublets, while the three right-handed charged lepton singlets are denoted                       |    |
|     | by $l, \omega$ denotes $e^{i\pi/3}$ , $T$ and $S$ are the symmetry transformation matrices for $Z_6$    |    |
|     | and $Z_2$ respectively  | 78 |
| 3.4 | The $Z_2 \times Z_6$ symmetry realization for the $C_{11}$ pattern within type-I seesaw scenario.       |    |
|     | $\Phi$ are five Higgs doublets, $D_L$ refers to the flavor three left handed lepton doublets,           |    |
|     | while the three right-handed charged lepton singlets are denoted by $l, \omega$ denotes $e^{i\pi/3}$ ,  |    |
|     | $T$ and $S$ are the symmetry transformation matrices for $Z_6$ and $Z_2$ respectively                   | 80 |
| 3.5 | The $Z_2 \times Z_6$ symmetry realization for the $C_{22}$ pattern within type-I seesaw scenario.       |    |
|     | $\Phi$ are five Higgs doublets, $D_L$ refers to the flavor three left handed lepton doublets,           |    |
|     | while the three right-handed charged lepton singlets are denoted by $l, \omega$ denotes $e^{i\pi/3}$ ,  |    |
|     | $T$ and $S$ are the symmetry transformation matrices for $Z_6$ and $Z_2$ respectively                   | 81 |
| 3.6 | The $Z_2 \times Z_6$ symmetry realization for the $C_{13}$ pattern within type-I seesaw scenario.       |    |
|     | $\Phi$ are five Higgs doublets, $D_L$ refers to the flavor three left handed lepton doublets,           |    |
|     | while the three right-handed charged lepton singlets are denoted by $l, \omega$ denotes $e^{i\pi/3}$ ,  |    |
|     | $T$ and $S$ are the symmetry transformation matrices for $Z_6$ and $Z_2$ respectively                   | 81 |