

بسم الله الرحمن الرحيم





شبكة المعلومات الجامعية التوثيق الالكتروني والميكرو فيلم



جامعة عين شمس

التوثيق الإلكتروني والميكرو فيلم

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**Bayesian analysis of a mixture of two components of
different families of distributions under different
censoring schemes**

A Thesis

**Submitted for the Award of the Ph.D. Degree in Science
in Mathematical Statistics**

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CONTENTS

	Page
ACKNOWLEDGEMENTS.....	i
ABSTRACT.....	ii
SUMMARY.....	iii

CHAPTER (I)

Introduction and Basic Concepts

1.1. Introduction.....	1
1.2. Basic Definition.....	2
1.3. Previous Studies.....	4
1.4. Basic Concepts.....	5
1.4.1. Maximum likelihood estimation.....	5
1.4.1.1. Asymptotic confidence intervals.....	7
1.4.2. Different censoring schemes	7
1.4.2.1. Complete sample.....	7
1.4.2.2. Type-I censoring.....	8
1.4.2.3. Type-II censoring.....	8
1.4.2.4. Hybrid censored scheme.....	8
1.4.2.5. Progressive Type-II censoring scheme.....	11
1.4.2.6. Progressive first failure censoring scheme.....	12
1.4.3. Basic definitions of Bayesian statistics.....	13
1.4.3.1. Prior distribution.....	13
1.4.3.2. Posterior distribution.....	14
1.4.3.3. Loss function.....	15
1.4.4. Monte Carlo methods.....	16
1.4.5. The approximation of Lindley.....	17
1.4.6. The approximation of Tierney and Kadane.....	18
1.4.7. Bayesian prediction.....	19

CHAPTER (II)

Bayesian Estimation and Prediction from a Mixture of Weibull and Lomax Distributions Based on Complete Sample

	Page
2.1. Introduction.....	20
2.2. Maximum Likelihood Estimation.....	21
2.3. Bayesian Estimation.....	22
2.3.1. The posterior distribution under the assumption of informative prior.....	22
2.3.1.1. Bayes estimator under squared error loss function.....	23
2.3.1.2. Bayes estimator under linear-exponential loss function (LINEX).....	24
2.3.1.3. Bayes estimator under general entropy loss function.....	24
2.3.2. The posterior distribution under the assumption of non-informative prior.....	24
2.3.2.1. Bayes estimator under squared error loss function.....	25
2.3.2.2. Bayes estimator under linear-exponential loss function (LINEX).....	25
2.3.2.3. Bayes estimator under general entropy loss function.....	26
2.4. Bayesian Prediction.....	26
2.5. Simulation Study.....	28
2.6. Numerical Example.....	29
2.7. Summary and Conclusion.....	30

CHAPTER (III)

Bayesian Estimation and Prediction Based on Progressively First Failure Censored Scheme from a Mixture of Weibull and Lomax Distributions

3.1. Introduction.....	49
3.2. Maximum Likelihood Estimation.....	50
3.3. Bayesian Estimation.....	52
3.3.1. Markov chain Monte Carlo method	54
3.4. Comparison Study.....	55
3.5. Bayesian Prediction.....	56
3.5.1. Numerical example.....	58
3.6. Conclusion.....	59

CHAPTER (IV)
Bayesian Approximation for a Mixture Weibull and Lomax Distributions
Based on Progressive Type-II Censoring Scheme

	Page
4.1. Introduction.....	68
4.2. Maximum Likelihood Estimation.....	68
4.3. Bayesian Estimation.....	71
4.3.1. Lindley's approximation method.....	72
4.3.2. Tierney_ Kadane's approximation method.....	81
4.4. Simulation Study.....	86
4.5. Conclusion.....	87

CHAPTER (V)
Parameter Estimation for a Mixture of Inverse Chen and Inverse
Compound Rayleigh Distributions Based on Type-II Hybrid Censoring
Scheme

5.1. Introduction.....	93
5.2. Maximum Likelihood Estimation.....	94
5.3. Bayesian Estimation.....	98
5.3.1. Bayes estimator under squared error loss function.....	99
5.3.2. Bayes estimator under linear-exponential loss function (LINEX).....	100
5.3.3. Bayes estimator under general entropy loss function.....	100
5.4. Credible Interval.....	101
5.5. Bayesian Two-Sample Prediction.....	101
5.6. Numerical Application.....	103
5.6.1. Comparison of estimation.....	103
5.6.2. Example.....	105
5.7. Conclusion.....	105

CHAPTER (VI)
Parameter Estimation for a Mixture of Inverse Chen and Inverse
Compound Rayleigh Distributions Based on Type-I Hybrid Censoring
Scheme

6.1. Introduction.....	121
------------------------	-----

	Page
6.2. Maximum Likelihood Estimation.....	121
6.3. Bayesian Estimation.....	124
6.3.1. Tierney_ Kadane's approximation method.....	125
6.4. Simulation Study.....	134
6.5. Conclusion.....	134
REFERENCES.....	141
ARABIC SUMMARY	

List of Tables

Table	Page
2.1. Average estimates and corresponding MSE of the parameter α_1 based on informative prior.....	32
2.2. Average estimates and corresponding MSE of the parameter α_2 based on informative prior.....	33
2.3. Average estimates and corresponding MSE of the parameter p based on informative prior.....	34
2.4. Average estimates and corresponding MSE of the parameter α_1 based on non- informative prior.....	35
2.5. Average estimates and corresponding MSE of the parameter α_2 based on non- informative prior.....	36
2.6. Average estimates and corresponding MSE of the parameter p based on non- informative prior.....	37
2.7. Bayes estimates and MSE under squared error loss function when $(\alpha_1, \alpha_2, p) = (1, 2, 0.45)$	38
2.8. Bayes estimates and MSE under LINEX Loss Function when $\alpha_1 = 1$	38
2.9. Bayes estimates and MSE under LINEX Loss Function when $\alpha_2 = 2$	39
2.10. Bayes estimates and MSE under LINEX Loss Function when $p = 0.45$	39
2.11. Bayes estimates and MSE under general entropy loss function when $\alpha_1 = 1$	40
2.12. Bayes estimates and MSE under general entropy loss function when $\alpha_2 = 2$	40
2.13. Bayes estimates and MSE under general entropy loss function when $p = 0.45$	41
2.14. The 90% Bayesian prediction bounds, length of the Bayesian prediction and their simulated coverage probability for Y_s based on informative prior.....	42

Table	Page
2.15. The 90% Bayesian prediction bounds, length of the Bayesian prediction and their simulated coverage probability for Y_s based on non-informative prior.....	43
2.16. The 95% Bayesian prediction bounds, length of the Bayesian prediction and their simulated coverage probability for Y_s based on informative prior.....	44
2.17. The 95% Bayesian prediction bounds, length of the Bayesian prediction and their simulated coverage probability for Y_s based on non-informative prior.....	45
2.18. Average estimates corresponding to real data set in case informative prior.....	46
2.19. Average estimates corresponding to real data set in case non-informative prior.....	46
2.20. Bayesian prediction bounds Y_s , length of the Bayesian prediction corresponding 90% in case informative prior for the real data set.....	47
2.21. Bayesian prediction bounds Y_s , length of the Bayesian prediction corresponding 90% in case non-informative prior for the real data set.....	47
2.22. Bayesian prediction bounds Y_s , length of the Bayesian prediction corresponding 95% in case informative prior for the real data set.....	48
2.23. Bayesian prediction bounds Y_s , length of the Bayesian prediction corresponding 95% in case non-informative prior for the real data set.....	48
3.1. Censoring schemes of progressively censored.....	55
3.2. Average estimates and the corresponding MSE based MLE.....	60
3.3. Average estimates and corresponding MSE of the parameter α_1 based on informative prior.....	61
3.4. Average estimates and corresponding MSE of the parameter α_2 based on informative prior.....	62

Table	Page
3.5. Average estimates and corresponding MSE of the parameter p based on informative prior.....	63
3.6. Average estimates and corresponding MSE of the parameter α_1 based on non-informative prior.....	64
3.7. Average estimates and corresponding MSE of the parameter α_2 based on non-informative prior.....	65
3.8. Average estimates and corresponding MSE of the parameter p based on non-informative prior.....	66
3.9. Two sample prediction intervals for the future observation Y_s in case informative prior.....	67
3.10. Two sample prediction intervals for the future observation Y_s in case non-informative prior.....	67
4.1. Censoring schemes of progressively censored.....	87
4.2. Average estimates and the corresponding MSE based MLE.....	88
4.3. Average estimates and corresponding MSE of the parameter α_1 based on informative prior.....	88
4.4. Average estimates and corresponding MSE of the parameter α_2 based on informative prior.....	89
4.5. Average estimates and corresponding MSE of the parameter p based on informative prior.....	90
4.6. Average estimates and corresponding MSE of the parameter α_1 based on non-informative prior.....	91
4.7. Average estimates and corresponding MSE of the parameter α_2 based on non-informative prior.....	91
4.8. Average estimates and corresponding MSE of the parameter p based on non-informative prior.....	92
5.1. Average estimates and the corresponding MSE based MLE.....	106
5.2. Average estimates and corresponding MSE of the parameter λ_1 based on informative prior.....	107

Table	Page
5.3. Average estimates and corresponding MSE of the parameter λ_2 based on informative prior.....	109
5.4. Average estimates and corresponding MSE of the parameter p based on informative prior.....	111
5.5. Average estimates and corresponding MSE of the parameter λ_1 based on non-informative prior.....	113
5.6. Average estimates and corresponding MSE of the parameter λ_2 based on non-informative prior.....	115
5.7. Average estimates and corresponding MSE of the parameter p based on non-informative prior.....	117
5.8. Estimated value of λ_1, λ_2 and p based on informative prior.....	119
5.9. Estimated value of λ_1, λ_2 and p based on non-informative prior...	119
5.10. 90% Interval estimates of λ_1, λ_2 and p	119
5.11. 95% Interval estimates of λ_1, λ_2 and p	120
5.12. Two sample prediction intervals for Y_S in case of informative prior.....	120
5.13. Two sample prediction intervals for Y_S in case of non-informative prior.....	120
6.1. Average estimates and corresponding MSE for parameter λ_1	136
6.2. Average estimates and corresponding MSE for parameter λ_2	137
6.3. Average estimates and corresponding MSE for parameter β_1	138
6.4. Average estimates and corresponding MSE for parameter β_2	139
6.5. Average estimates and corresponding MSE for parameter p	140

List of Figures

Figure	Page
1.1. Schematic illustration of the hybrid censoring scheme.....	10

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ABSTRACT

ABSTRACT

MARWAH AHMED MOHAMED AEFA

Bayesian analysis of a mixture of two components of different families of distributions under different censoring schemes

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Finite mixtures models have received considerable attention in areas of survival analysis and reliability in recent years, from analysis in both the methodological development and multifarious applications. This thesis discuss the statistical inference of heterogeneous population model by using two component mixture model when the data are of different censoring schemes. We consider the maximum likelihood estimation and Bayes estimation of parameters assuming informative and non-informative priors under symmetric and asymmetric loss functions. In some cases three different approximation methods are used for Bayesian computation, importance sampling method, Lindley approximation and Tierney and Kadane approximation. We perform Monte Carlo simulation to compare the performance of the different methods. The Bayes prediction intervals are also determined.

Key Words: Mixture model, Progressive first failure censored scheme, Progressive Type-II censoring, Hybrid censored, Maximum likelihood estimation, Bayesian estimation, Bayesian prediction.

SUMMARY

SUMMARY

The survival analysis of lifetime (failure time) data is an important topic in statistics and has applications in different fields of life and also is a collection of statistical procedures for data analysis for which the outcome variable of interest is the time until an event occurs. Various parametric families of models are used in the analysis of lifetime data. For this purpose, we use a mixture model of Weibull and Lomax distributions and Inverse Chen and Inverse Compound Rayleigh distributions.

Finite mixture models provide a flexible framework to handle heterogeneous data supposed to comprise a number of subpopulation mixed in an unknown proportion, also to be of considerable interest in terms of both of the methodological development and multifarious applications. Mixture distributions have gained great interest for the analysis by many statisticians and mathematicians. For the purpose of the discussion, the estimation of surveys and their application in different fields of life, mixture distributions play a vital role in many practical applications. Direct applications of finite mixture models are medicine, botany, life testing, reliability, etc. Indirect applications include outliers, cluster analysis, latent structure models, modelling prior densities, empirical Bayes method and nonparametric density estimation.

Point estimation is one of the core topics in mathematical statistics. The Bayes methods consider the most common method of point estimation, and is an important approach to statistics, which formally seeks use of prior information and Bayes theorem to provide the formal basis for using this information. In recent years analysis of mixture models under Bayesian framework has received considerable attention.

This thesis focuses on the problem of estimating the parameters and prediction intervals under two-component mixture of different distributions under different censoring techniques and using different loss function.

The thesis consists of six chapters as follows:

Chapter I: Contains the concepts and basics of this thesis. Some of the previous studies of distributions used are presented.