



شبكة المعلومات الجامعية  
التوثيق الإلكتروني والميكروفيلم

# بسم الله الرحمن الرحيم



**MONA MAGHRABY**



شبكة المعلومات الجامعية  
التوثيق الإلكتروني والميكرو فيلم



# شبكة المعلومات الجامعية التوثيق الإلكتروني والميكرو فيلم



**MONA MAGHRABY**



شبكة المعلومات الجامعية  
التوثيق الإلكتروني والميكروفيلم

# جامعة عين شمس

## التوثيق الإلكتروني والميكروفيلم

### قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها  
علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات



### يجب أن

تحفظ هذه الأقراص المدمجة بعيدا عن الغبار



**MONA MAGHRABY**



Ain Shams University  
University College for Women  
(Arts, Science, and Education)  
Department of Mathematics

# Some Statistical Aspects for Non-linear Interaction Between Some Quantum Systems

A THESIS  
SUBMITTED IN PARTIAL FULFILLMENT OF REQUIREMENTS  
FOR THE DEGREE OF  
MASTER OF SCIENCE (M.Sc.)  
(APPLIED MATHEMATICS)

BY

**Asmaa Tarek Mohamed Abd-El latef**

Department of Mathematics  
University College for Women  
Ain Shams University

## SUPERVISORS

**Prof. Dr. Mohamed M. Ali Ahmed**  
Professor of Applied Mathematics  
Faculty of Science  
Al-Azhar University

**Prof. Dr. Samia Saeed Al-Azab**  
Professor of Applied Mathematics  
Faculty of Women (Science)  
Ain Shams University

**Ass. Prof. Doaa Ahmed M. Abu-kahla**  
Doctor of Applied Mathematics  
Faculty of Education  
Ain Shams University

( 2021 )



Ain Shams University  
University College for Women  
(Arts, Science, and Education)  
Department of Mathematics

M.Sc. Thesis  
(APPLIED MATHEMATICS)

Title of Thesis:

## **Some Statistical aspects For Non-linear Interaction between Some Quantum Systems.**

Thesis supervisors:

**Prof. Dr. Mohamed M. Ali Ahmed**

Professor of Applied Mathematics  
Faculty of Science  
Al-Azhar University

**Prof. Dr. Samia Saeed Al-Azab**

Professor of Applied Mathematics  
Faculty of Women (Science)  
Ain Shams University

**Ass. Prof. Doaa Ahmed M. Abu-kahla**

Doctor of Applied Mathematics  
Faculty of Education  
Ain Shams University



Ain Shams University  
University College for Women  
(Arts, Science, and Education)  
Department of Mathematics

## COURSES

The student has passed the following courses in partial fulfillment of the requirement for an M. Sc. degree:

- |                                                  |             |
|--------------------------------------------------|-------------|
| 1. Quantum Theory.                               | 3h per week |
| 2. Complex Analysis.                             | 3h per week |
| 3. Numerical Analysis.                           | 3h per week |
| 4. Writing Scientific Research.                  | 3h per week |
| 5. Partial Differential Equations.               | 3h per week |
| 6. Computational Methods in Applied Mathematics. | 3h per week |
| 7. Relativity Quantum Mechanics.                 | 3h per week |
| 8. Ethics of Scientific Research.                | 3h per week |

Head of Mathematics Department





قَالُوا سُبْحَانَكَ لَا عِلْمَ لَنَا إِلَّا مَا عَلَّمْتَنَا إِنَّكَ أَنْتَ الْعَلِيمُ الْحَكِيمُ



(سورة البقرة: 32)

# Contents

<b>Contents</b>	<b>ii</b>
<b>Abstract</b>	<b>v</b>
<b>List of Figures</b>	<b>vii</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Literature Review . . . . .	1
1.2 Overview of This Thesis . . . . .	5
1.3 The Hamiltonian . . . . .	6
1.3.1 The Pauli operators . . . . .	7
1.3.2 The collective angular momentum operators . . . . .	8
1.4 The Schrodinger Pictures of Quantum Systems . . . . .	9
1.5 The Rotating Wave Approximation (RWA) . . . . .	10
1.6 The Kerr-Like Medium Effect . . . . .	12
1.7 The Atomic Population Inversion . . . . .	13
1.8 The Normalized Second-Order Correlation Function . . . . .	14
<b>2 Analytical Solution of a Three-Level Atom Interacting with Four Systems of N-Two Level Atoms</b>	<b>17</b>
2.1 The Model Hamiltonian . . . . .	18



2.2	The Atomic Population Inversion . . . . .	23
2.3	The Correlation Function . . . . .	26
2.4	Conclusion . . . . .	28
<b>3</b>	<b>The interaction between a three-Level Atom and Four Systems</b>	
	<b>of N-Two Level Atoms with Kerr-like medium</b>	<b>37</b>
3.1	Introduction . . . . .	37
3.2	The Model . . . . .	39
3.3	The Atomic Inversion . . . . .	45
3.4	The Correlation Function . . . . .	47
3.5	conclusion . . . . .	50
<b>4</b>	<b>Different Interaction Between a Three-Level Atom and Four Sys-</b>	
	<b>tems of N-Two Level Atoms</b>	<b>61</b>
4.1	The Model . . . . .	62
4.2	The Atomic Population Inversion . . . . .	66
4.3	The Correlation Function . . . . .	68
4.4	Conclusions . . . . .	70
<b>5</b>	<b>Conclusion</b>	<b>77</b>
	<b>Appendix A</b>	<b>81</b>
	<b>Appendix B</b>	<b>87</b>
	<b>Bibliohraphy</b>	<b>90</b>

# Acknowledgment

In the name of Allah, the Merciful, the Beneficent. Praise be to Allah, the Lord of all worlds. Prayers and peace be upon our Prophet, Muhammad, his family, and all of his companions.

First and foremost, my limitless thanks to ALLAH , for help, blessing, and showing me the right path to introduce this work. I am totally sure that this work would have never come to light, without ALLAH almighty.

All my profound gratitude goes to Prof. Abdel-Shafy F. Obada, and to my supervisors Prof. Mohamed M. A. Ahmed, Prof. Samia Saeed Al-Azab, and special thanks to Dr. Doaa A. M. Abu-kahla.

My completion of this project could not have been accomplished without their inspiring guidance, encouragement, advice, valuable discussion, and the limitless time and effort they put into researching various topics, answering my many questions and proof reading several times of this thesis. I couldn't imagine having better supervisors and mentors to my thesis study.

All members of the "Quantum seminar" in Al-Azhar university have been brilliant, friendly, and patient collaborators during our studying. I wish to extend my

thanks to Mathematics Department, my Professors, my colleagues and Faculty of Science, University college for women - Ain shams University.

Last and not least, I would like to thank my family for all their support. I am deeply indebted to my parents who raised me with love and encouragement.

Thank you

Cairo, Egypt  
*2021*

*Asmaa Tarek.*  
*Asmaa.Tarek@women.asu.edu.eg*

# Abstract

In this thesis, the interaction between a three-level atom and four systems of N-two-level atoms in different cases is studied. Besides, the atom is coupled with the systems by time-dependent coupling parameters.

The analytical solution of this model is presented. further, the influence of the Kerr-like medium on this model is introduced. The approximate solution of this system and the wave function are obtained, under a specific conditions, and therefore the Schrodinger equation is solved.

Finally, we consider our model, but in different interaction Hamiltonian. The exact solution for the wave function is obtained, by taking some special cases. We investigate some physical statistical properties of the three systems such as the atomic population inversion and the second-order correlation function.

Then also the effects of several parameters such as the numbers of two-level atoms of the four systems and the coupling parameters between spins on these statistical properties are studied.

The analysis of results manifests that correlated behavior can be controlled by

adjusting some parameters like the coupling parameters between spins.

Also, it has been revealed that the atomic inversion and correlation function are greatly affected by the Kerr medium.

# List of Figures

- 2.1 Scheme of the interaction between a three-level atom coupled to four systems of N-two level atoms . . . . . 19
- 2.2 The evolution of the atomic inversion as function of scaled time  $t$ .  $\Delta = 0, m_1 = m_2 = m_3 = m_4 = 1, \beta_1 = \beta_2 = 1, j_1 = 30, j_2 = 50, j_3 = 20, j_4 = 40$  and the initial state is  $|\Psi(0)\rangle_s = |1\rangle$ , Where the thick red, black, dashed blue curves correspond, respectively, to  $\lambda_1 = \lambda_2 = 0.25, 0.5, 1$ . . . . . 31
- 2.3 Figures of the case in which  $\Delta = 0, \lambda_1 = \lambda_2 = 1, \beta_1 = \beta_2 = 1$  and  $j_1 = 30, j_2 = 50, j_3 = 20, j_4 = 40$ . The bold blue, red, dashed black curves correspond, respectively, to  $m_1 = m_2 = m_3 = m_4 = 1, 12, 14$ , where the initial state is (a)  $|\Psi(0)\rangle_s = |1\rangle$ , (b)  $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}} |1\rangle + \frac{1}{\sqrt{2}} |2\rangle$  and (c)  $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}} |1\rangle + \frac{1}{2} |2\rangle + \frac{1}{2} |3\rangle$ . 32



- 2.4 Figure of the case in which  $\Delta = 0$ ,  $\lambda_1 = \lambda_2 = 1$ ,  $m_1 = m_2 = m_3 = m_4 = 1$ ,  $j_1 = 30$ ,  $j_2 = 50$ ,  $j_3 = 20$ ,  $j_4 = 40$ . The thick, green, dashed black curves correspond, respectively, to  $\beta_1 = \beta_2 = 700, 800, 1000$ , where the initial state is (a)  $|\Psi(0)\rangle_s = |1\rangle$ , (b)  $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}} |1\rangle + \frac{1}{\sqrt{2}} |2\rangle$  and (c)  $|\Psi(0)\rangle_s = \frac{1}{\sqrt{2}} |1\rangle + \frac{1}{2} |2\rangle + \frac{1}{2} |3\rangle$ . . . . . 33
- 2.5 The normalized second-order correlation functions  $g_1^2(t)$ ,  $g_2^2(t)$ ,  $g_3^2(t)$  and  $g_4^2(t)$  against the time  $t$ .  $\Delta = 0$ ,  $m_1 = m_2 = m_3 = m_4 = \frac{3}{2}$ ,  $\beta_1 = \beta_2 = 1$ ,  $j_1 = j_2 = j_3 = j_4 = \frac{5}{2}$  and the initial state is  $|\Psi(0)\rangle_s = |1\rangle$ , where the dashed black, red, thick blue curves correspond, respectively, to  $\lambda_1 = \lambda_2 = 0.25, 0.5, 1$ . . . . . 34
- 2.6 The normalized second-order correlation functions  $g_1^2(t)$ ,  $g_2^2(t)$ ,  $g_3^2(t)$  and  $g_4^2(t)$  against the time  $t$ .  $\Delta = 0$ ,  $\lambda_1 = \lambda_2 = 1$ ,  $\beta_1 = \beta_2 = 1$ ,  $j_1 = j_2 = j_3 = j_4 = \frac{5}{2}$  and the initial state is  $|\Psi(0)\rangle_s = |1\rangle$ , where the thick green, dot dashed blue, red, dashed black curves correspond, respectively, to  $m_1 = m_2 = m_3 = m_4 = \frac{-3}{2}, \frac{-1}{2}, \frac{1}{2}, \frac{3}{2}$ . . . 35
- 2.7 The normalized second-order correlation functions  $g_1^2(t)$ ,  $g_2^2(t)$ ,  $g_3^2(t)$  and  $g_4^2(t)$  against the time  $t$ .  $\Delta = 0$ ,  $m_1 = m_2 = m_3 = m_4 = \frac{3}{2}$ ,  $\lambda_1 = \lambda_2 = 1$ ,  $j_1 = j_2 = j_3 = j_4 = \frac{5}{2}$  and the initial state is  $|\Psi(0)\rangle_s = |1\rangle$ , where the thick green, red, dot dashed blue, dashed black curves correspond, respectively, to  $\beta_1 = \beta_2 = 1, 3, 5, 7$ . . . . . 36