شبكة المعلومات الجامعية
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بسم الله الرحمن الرحيم

HANAA ALY

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جامعة عين شمس

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HANAA ALY

Faculty of Education
Mathematics Department

Haar Wavelet Spectrum and Statistics of a Pulsed-Driven Harmonic Oscillator

A Thesis

Submitted in Partial Fulfillment of the Requirements of the Master's Degree in Teacher's Preparation of Science in Applied Mathematics

(Quantum Mechanics)

Submitted to:

Department of Mathematics, Faculty of Education,
Ain Shams Universty

By

Hager Mamdouh Ibrahim Atalla

Demonstrator at
Department of Mathematics, Faculty of Education,
Ain Shams University

Supervised by

Prof. Galal Mahrous Moatimid
Professor of Applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

Dr. Osman Mohamed Frege
Associate Professor of Applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

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Supervisors:

No.	Name	Profession	Signature
1.	**Prof. Galal Mahrous Moatimid**	Professor of Applied Mathematics, Department of Mathematics, Faculty of Education, Ain Shams University.	
2.	**Dr. Osman Mohamed Frege**	Associate Professor of Applied Mathematics, Department of Mathematics, Faculty of Education, Ain Shams University.	

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Abstract

In this thesis, the Haar wavelet spectrum and statistics of a pulsed-driven harmonic oscillator (HO) are studied. The Heisenberg model equations of a single HO driven by a laser pulse of arbitrary shape are solved. The mean photon number is calculated for various shapes of laser pulses (rectangular, triangular, symmetric square and anti symmetric square pulses) and for different initial states of the HO in (vacuum, number and coherent states).

Sub-Poissonian photon statistics of the emitted radiation by a laser pulsed driven non-dissipative HO is investigated analytically through Mandel's Q_M parameter. Conditions for sub-Poissonian behaviour are derived for different initial states of the HO (namely, the number state, single mode squeezed vacuum state, single mode squeezed coherent state and single mode squeezed thermal state) and for arbitrary pulse shape. Computational results are presented for various pulse shapes (namely, the rectangular, triangular, symmetric square and anti-symmetric square pulses). It is also shown that the driven HO with initial coherent, number and squeezed vacuum states evolves in time to a corresponding state, respectively, but with the HO operators and the vacuum state are displaced depending on the input pulse shape.

The transient scattered spectrum with Haar wavelet window function is derived analytically for a single quantized HO interacting with the same laser pulses mentioned above. Effects of the window function parameters (delation and shift parameter) are examined if the HO was initially in vacuum, number and coherent states.

Contents

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Introduction

Chapter 1

Introduction

1.1 Thesis Outline

The thesis consists of four chapters. Their contents are listed as:

Chapter 1
This chapter is an introduction for the thesis in which basic concepts and information needed for the following chapters are presented. First, briefs on quantum mechanics and quantum optics are introduced. Second, states of the quantized radiation field are reviewed. Third, the definitions of the Fourier transform (FT) and wavelet transform (WT) are given. Fourth, the concepts of photon statistics and the Mandel Q_M parameter are outlined. Finally, the master equation of non-dissipative HO interacting with arbitrary laser pulse is presented.

Chapter 2
In this chapter, Heisenberg model equations of a single non-dissipative HO driven by a laser field of arbitrary shape are solved. The solution obtained are used to calculate the mean photon number for various shapes of laser pulses (namely, rectangular, triangular, symmetric and anti-symmetric square pulses) if the HO was initially in vacuum, number and coherent states.

Chapter 3
Sub-Poissonian photon statistics of emitted radiation by a laser pulsed driven non-dissipative HO is investigated analytically through Mandel's Q_M parameter. Conditions for sub-Poissonian behaviour are derived for different initial states of the HO (namely, the number state $|n_o\rangle$, and single mode

squeezed vacuum state $|\xi\rangle$, single mode squeezed coherent state $|\xi, \alpha\rangle$, single mode squeezed thermal state of density operator $\rho(\xi, \bar{n})$) and for arbitrary pulse shape. Computational results are presented for four examples of pulse shapes (namely, rectangular, triangular, symmetric and antisymmetric square pulses). It is also shown that, the driven HO with initial coherent, number and squeezed vacuum states evolves in time to a corresponding state, respectively, but with the HO operators and the vacuum state are displaced, depending on the input pulse shape.

Chapter 4
The transient scattered spectrum with Haar wavelet window function is derived analytically for a single quantized non-dissipative HO interacting with various shapes of laser pulses (rectangular, triangular, symmetric and anti symmetric square pulses). Effects of the window function dilation and shift parameter are examined if the HO was initially in vacuum, number or coherent states.

1.2 Quantum Mechanics

Classical mechanics is the theory which explains matter and energy in the macroscopic range. This theory based on Newton's laws and Maxwell's electromagnetic theory. By contrast, the quantum mechanics is the theory which explains the behaviour of matter and its interaction with energy on the scale of atomic and subatomic particles (microscopic range) [1].

The need to quantum mechanics appeared as a result of the failure of the classical theory in explaining various microscopic phenomena, such as, the black body radiation, the photoelectric effect, atomic spectra and electron diffraction [1].

In 1900 Max Planck introduced the concept of the quantum of energy, as the first penetration to the classical theory, and gave an accurate explanation of the black body radiation. His main assumption was that the energy exchange between matter and radiation occurs in discrete amounts. He argued that the energy exchange between matter and an electromagnetic wave of frequency ν takes place only in integer multiples of $\hbar\nu$, where $\hbar$ is the reduced Planck's constant [1–6].

Albert Einstein in 1905 assumed that, following Planck's idea, light itself is made of discrete bits of energy called photons each of energy $\hbar\nu$, where ν is the frequency of the light. This assumption enabled him to give an accurate explanation to the photoelectric effect. He showed that the kinetic energy of the ejected electron depends on the frequency of the incident radiation not on its intensity [1, 3–6].

The first complete picture of quantum mechanics, Heisenberg picture (Matrix mechanics), was introduced by Heisenberg in 1925. Within a year, Schrödinger introduced the wave mechanics, Schrödinger picture, and showed that it is equivalent to matrix mechanics. After that, Dirac came up with transformation mechanics (kets and bras) and showed that both matrix and wave mechanics are special cases of it [1, 6].

1.3 Quantum Optics

Quantum optics is the branch of quantum mechanics, which aims to study matter-radiation (light) interaction and radiation-radiation interaction. Two approaches are used in quantum optics namely, the semi-classical and quantum mechanical approaches. In the first, light is treated as a classical electromagnetic wave and the atom is treated quantum mechanically. This approach is suitable to study some phenomena like absorption of light by atoms. In the second, both atom and light are treated quantum mechanically. This approach is required to explain some phenomena such as spontaneous emission and Lamb shift [7, 8]

Quantum optics is helpful in electron microscopy which is applied in hospitals for surface scanning of burnt layer of skin. Similarly, the functioning of Burglar alarms, fire alarms and street light switches can be modified efficiently with the help of quantum optics. The principles of quantum optics can also be applied in photography, TV camera, TV receivers, Cinematography, automatic doors, night vision scopes etc. They are also used to measure the transparency or opacity of liquids and solids, to study the spectrum of celestial bodies and their temperature and control heat in chemical reactors [9].

1.4 States of the Radiation Field

One of the most important examples in quantum mechanics is the HO. It is an example of the few quantum systems for which an analytical exact solution is found [10–12]. The HO plays a significant role in theory of lattice vibrations in solids, quantum theory of electromagnetic fields and laser theory.

Classically, the hamiltonian of the HO of unit mass in one dimension is given by,

$$H = \frac{1}{2}\Big(p^2 + \omega^2 x^2\Big), \tag{1.1}$$

where x, p and ω are the coordinate, the momentum and the frequency of the HO, respectively. Using the classical Hamilton equations of motion,

$$\begin{aligned} \dot{x}(t) &= \frac{\partial H}{\partial p} = p \\ \dot{p}(t) &= -\frac{\partial H}{\partial x} = -\omega^2 x \end{aligned} \tag{1.2a}$$

of solutions,

$$\begin{aligned} x(t) &= x(0) cos\omega t + \frac{p(0)}{\omega} sin\omega t, \\ p(t) &= \omega x(0) sin\omega t + p(0) cos\omega t. \end{aligned} \tag{1.2b}$$

Introducing the complex variable,

$$\hat{a} = \frac{1}{\sqrt{2\omega}}\Big(\omega x + ip\Big), \tag{1.3}$$

and using the solution (1.2b) gives,

$$\hat{a}(t) = \hat{a}(0) e^{-i\omega t}. \tag{1.4}$$

The Hamiltonian (1.1) may be rewritten in terms of $\hat{a}$ and its complex conjugate $\hat{a}^*$ as,

$$H = \omega \hat{a}^* \hat{a}, \tag{1.5}$$

which is simpler than that in terms of x and p. In quantum mechanics, hermitian operators $\hat{x}$, $\hat{p}$ and $\hat{H}$ are associated with the dynamical variables x, p and H assuming that $\hat{x}$ and $\hat{p}$ satisfy the commutation relation,

$$\Big[\hat{x}, \hat{p}\Big] = i\hbar, \tag{1.6}$$

where $\hbar$ is planck's constant divided by 2π.
The corresponding Hamiltonian operator $\hat{H}$ to Eq. (1.1) is rewritten as,

$$\hat{H} = \frac{1}{2}\Big(\hat{p}^2 + \omega^2 \hat{x}^2\Big). \tag{1.7}$$