

بسم الله الرحمن الرحيم





شبكة المعلومات الجامعية التوثيق الالكتروني والميكروفيلم



جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

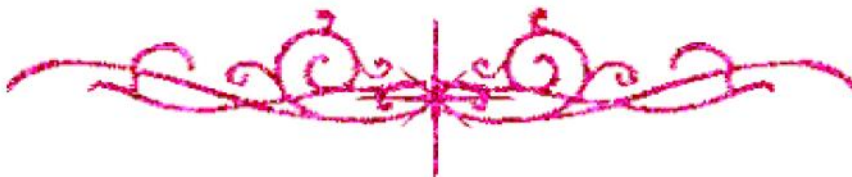
قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها
علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات



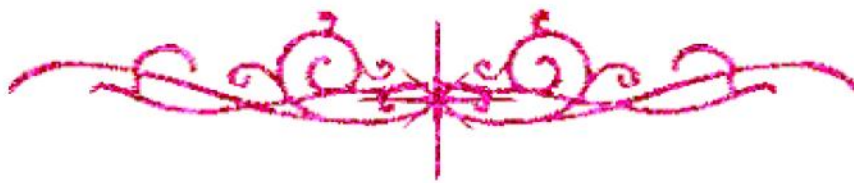
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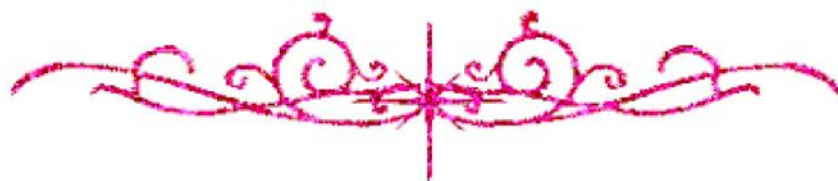


بعض الوثائق الأصلية تالفة





بالرسالة صفحات
لم ترد بالأصل





Faculty of Education
Mathematics Department

SET THEORY'S EXTENSIONS-BASED ANALYSIS

A Thesis

Submitted in Partial Fulfillment of the Requirements of the Doctor of
Philosophy Degree in Teacher's Preparation in Science

(Pure Mathematics)

Submitted to:

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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”BISMILLAH ERRAHMAN ERRAHEEM”**

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Hanan Sakr

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Dedication

Dedicated to,

My husband

&

My children;

Alaa and Youssef

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