شبكة المعلومات الجامعية
التوثيق الإلكتروني والميكروفيلم

بسم الله الرحمن الرحيم

MONA MAGHRABY

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قسم

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MONA MAGHRABY

THE SPECIAL LAW OF VARIATION FOR HUBBLE'S PARAMETER IN COSMOLOGICAL MODELS AND THE PRESENT UNIVERSE

A THESIS

In Partial Fulfillment for the Requirement of the Degree of Master for Teacher Preparation of Science in Applied Mathematics

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Aryn T. Shafeek

Abstract

The current study mainly deals with the development and establishment of some mathematical cosmological models for interpretation of the origins and evolution of the universe according to modern observations.

In chapter one, we stated a literature review, and some of work done in this field.

In chapter two, we propose two cosmological models which can explain the evolution of the universe after the Big Rip moment. The first model is the periodic universe with a varying deceleration parameter of the second degree of cosmic time. The second model is generated by utilizing a varying deceleration parameter as a polynomial of degree $l-1$ in the cosmic time.

In chapter three, we construct a gravitational field with torsion by using the parameterized absolute parallelism geometry. Under the influence of the gravitational field with torsion, the cosmological models are obtained and discussed.

In chapter four, we derive a path deviation equation in the absolute parameterized parallelism geometry. This chapter tackles the stability and singularity of cosmological models from the perspective of Parameterized absolute parallelism geometry.

List of publications

1- M. A. Bakry, and Aryn T. Shafeek. "The periodic universe with varying deceleration parameter of the second degree." *Astrophysics and Space Science* **364**. 8 (2019): 1-6.

2- M. A. Bakry, G. M. Moatimid, and A. T. Shafeek. "Cosmological models generated by utilizing a varying polynomial deceleration parameter." *Indian Journal of Physics* (2021): 1-18.

3- M. A. Bakry, and Aryn T. Shafeek. "Big Rip and Big Crunch Cosmological Models in a Gravitational Field with Torsion." *Gravitation and Cosmology* **27**. 1 (2021): 89-104.

SUMMARY

SUMMARY

The current study mainly deals with the development and establishment of some mathematical cosmological models for interpretation of the origins and evolution of the universe according to modern observations. In the present work, some current cosmological models are investigated and developed in such a way that would help to predict the physical behavior of the universe in the distant future. The thesis consists of four chapters, 52 figure, 24 tables and ends with a list of references.

CHAPTER I briefly reviews three geometric structures. The first structure is Riemannian geometry by means of which the field equations are described in general relativity theory, as well as geodesic equations. Also, a brief summary of the Raychaudhuri equation is presented. The second structure is Absolute parallelism space (AP-Space). The third one is the Parameterized absolute parallelism geometry by means of which it was possible to deduce the modified Raychaudhuri equation which will be used to study the singularity of the cosmological models in chapter IV. At the end of chapter I, a brief review of some of the previous cosmological models that appeared before 1998 and after 1998 is outlined.

CHAPTER II proposes two cosmological models which can explain the evolution of the universe after the Big Rip moment. The first model is the periodic universe with a varying deceleration parameter of the second degree of cosmic time, according to which the universe passes through the Big Rip and then retreats as it was at the beginning. According to this model, a closed, flat, and open universe is allowed. The second model is generated by utilizing a varying deceleration parameter as a polynomial of degree $l-1$ in the cosmic time. The Milne model, the Radiation model, the Einstein-de Sitter model and the Einstein model are concluded as a special case at $(l=1)$. The linearly varying deceleration parameter model is obtained at $(l=2)$. Also, the periodic universe with a varying deceleration parameter of the second degree model is found at $(l=3)$. This model covers all of the previous models and covers the periodic models at $(l>3)$. From the previous analysis of the varying polynomial deceleration parameter, one can observe that by increasing the degree of the deceleration parameter, a periodic multi-stage universe can be obtained. It starts with the Big Bang, passes through the Big Rip, and then retracts to its first state in a repeated operation. Further the universal model PUVDP is compared with LVDP, PVDP, CDP and ΛCDM models.

CHAPTER III constructs a gravitational field with torsion by using the parameterized absolute parallelism geometry. Under the influence of the gravitational field with torsion, the cosmological models are obtained and discussed. It is to be noted that the torsion term leads to inflationary cosmological models with a constant deceleration parameter. We suggest a special law for the deceleration parameter to study the big rip model. The closed and open models turn out to be impossible since the condition of the energy density $\rho \geq 0$ is unreliable, whereas the flat model is

possible given the fact that condition $\rho \geq 0$ is satisfied in this model. Also, the big crunch model is suggested which passes through two stages. In the first stage, the universe starts with the Big Bang, then reaches the Big Rip; while in the second stage the universe begins with the Big Rip until it reaches the Big Crunch moment. The torsion plays a relevant role in the second half-age of the universe. Also, this chapter presents observational bounds on cosmic torsion.

CHAPTER IV derives a path deviation equation in the absolute parameterized parallelism geometry. This chapter tackles the stability and singularity of cosmological models from the perspective of Parameterized absolute parallelism geometry. Therefore, the aim of this chapter is to examine the effect of the torsion term on the stability and the singularity of the cosmological models. It is to be noted that the torsion term does not affect the stability/instability of the cosmological models. The universe starts with the stable case at the Big Bang, then it gradually moves to the instability case.

On the other hand, in Riemannian geometry without torsion term, the universe begins with a singularity at the Big Bang, and ends with a non- singularity at the Big Rip. Conversely, in the Parameterized absolute parallelism geometry with the torsion term, the universe begins with a non-singularity at the Big Bang, and ends with singularity at the Big Rip.

CONTENTS

CHAPTER I: Introduction

CHAPTER II: Variation of Hubble Parameter for Cosmological Models with Varying Deceleration Parameter of the Higher Degree ot the Cosmic Time

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