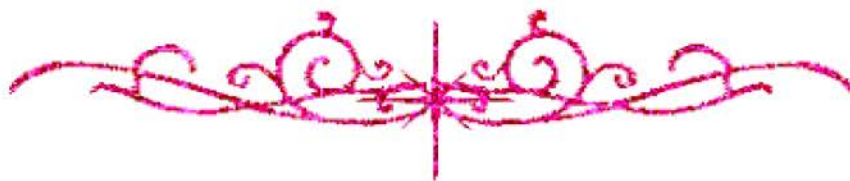


# بسم الله الرحمن الرحيم





# شبكة المعلومات الجامعية التوثيق الالكتروني والميكرو فيلم





# جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

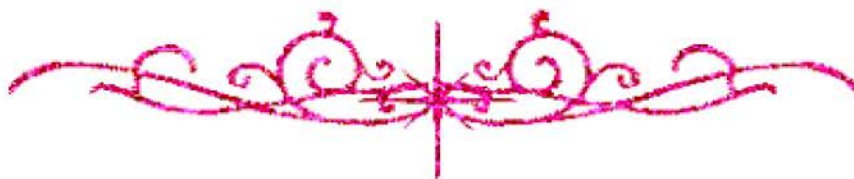
## قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها  
علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات



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تحفظ هذه الأقراص المدمجة بعيدا عن الغبار





القسم: الرياضيات

## **Behavior of Strategies in Three–Player Iterated Game**

**A Thesis submitted in Partial Fulfillment of the  
requirement for the Master Degree in Science in Pure  
Mathematics**

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## SUMMARY

This thesis provides important insights into game theory. Game theory enters our everyday life and treatment. There is no successful business without cooperation. Part of our life requires cooperation with each other and the other part is struggle and competition to reach the best. The study of game theory makes players make the best choices considering certain conditions to obtain results. Each player chooses his strategy without looking at the other player. Each player gets the same payoff as the other player if they choose the same strategy in single community games. In two population game, the payoff will be different and unequal. This thesis consists of three chapters on several topics in game theory.

The first chapter is an introduction to the thesis and contains many important and basic terms in game theory, an example of this is the concept of the game and its content and examples of games, including the prisoner's dilemma and the most important games in game theory. Other very important games in game theory are mentioned. It also contains the concept of Nash equilibrium, which is a very important concept for game theory.

The second chapter concerned with two population game. The prisoner's dilemma was explained more broadly in terms of two or three players. It contains the most basic definitions that we need in game theory. We also talked about cooperation and likened it to human cells and mentioned three types. There is direct cooperation such as the

exchange of goods and the obtaining of money by one party which is considered indirect cooperation.

The third chapter focused on the prisoner's dilemma in three players, and the players were divided into two groups, "Tempered players and Natural-Tempered players". We used a programming language to get the best strategy for the players in each group. We collected similar strategies in these two groups and studied if an individual meets all the players in his group, what is the best strategy in this group. The results of this chapter were published in **Journal of Mathematical and Computational Science in 2021** under name El-seidy, Essam, Reda, Naglaa M and El-Salam, Salsabeel M A B D, **Predicting mood behaviors of prisoner's dilemma players (2021), 4650-4667, 11(4).**

## **CHAPTER (I)**

### **DEFINITIONS AND BASIC CONCEPTS**

#### **1.1. Introduction**

We are humans can't live without each other, and surprisingly, sometimes production and exchange require some degree of cooperation between individuals, but relationships can also lead to destructive conflicts. We seem to have survived those relationships. Human history is both a history of wars and a history of successful cooperation. Many human relationships have the potential for cooperation and communication, as well as conflict and danger. There are many examples of relationships between husband and wife, brothers, countries, administrative, trade unions, neighbours, students, and teachers, etc [1].

All over the world people interact which sometimes is collaborative, for example when collaborators are successfully collaborating on a project. In other cases, the relationship is competitive, for example, two or more companies are competing for market share. In each case, they take it over, despite the obstacles that are hard to imagine and is called a strategic position because we have to think about how others around us choose their actions.

Parlor games were studied early in an attempt to formulate optimal strategies. In 1713, James Waldegrave solved a particular card game and sent it to his colleagues Nicolas Bernoulli and Pierre-Rémond de Montmort which coincides with modern theory. In the nineteenth

century, the equilibrium models of oligopoly were explored by Augustin, and bargaining problems were addressed in the context of exchange in economics.

In 1913, the first formal theorem for games (a result of chess) was proved. Emile Borel gave the concept of a strategy then, in the 1920s, 1930s, and 1940s, John von Neumann gave a fairly remarkable work, presenting a real and rigorous general theory of strategic positions called game theory. Von Neumann and Oskar Morgenstern explicate Game Theory, which explains how games are represented mathematically accurately and behaviour analysis can be applied to a small class of strategy. The game theory became a very important and wonderful work with the work of John Nash in the middle of the century and the creation of concepts between non-cooperative and "cooperative" theoretical models.

In 1950, John Nash explained that limited games have a point of equilibrium, where by each player chooses the best option for him in light of the other players' choices. In the 1950s and 1960s, game theory was applied to problems of politics and war in theory. Additionally, it has found applications in evolution, biology, sociology, and psychology. In 1994 Nash, John Harsanyi, and Reinhard Selten won the Nobel Prize in Economics. Historically, developments in game theory have been economics, but the game theory has featured interesting applications in fields as diverse as biology, computer science, social science, and business.



## **1.2. What is game theory?**

Game theory is a combination of mathematical theory. Mathematics was translated through game theory into language theory by Newman. The common language to write about this game is competition, an analytical resource that can be used to explain the game theory, current behaviour to be explained, or business strategies. It is used in particular in the sciences to analyse long-term situations such as biology or sociology. For example, there may be situations with an animal in which cooperation has developed for mutual benefit.

The game is a complete set of rules. Everyone in it is called a player. The player makes a decision called a move or an action. The action is not the same as the strategy. The strategy is the plan of the game. It is the move that the player chooses in all possible positions. Every player in the game must be a rational player. Which means that they try to win the game and optimize for the payoffs. The others are also aware of their optimal payoffs.

The game has some rules, players and some strategies which has at least two players. The players make decision. It isn't necessary that the player is single person. The players may be producers, organization or nation. The strategy is the action from players. The player's feasible strategies are the rules. Every player has the number of strategies which is finite. They are known by all players. There are two types of strategies are the pure strategy and mixed strategy. The moves of game consist of simultaneous game or sequential game. Each player in the game doesn't know the reply of the other players. Every player has payoff after the game is over which depends on the motive of game.

The returns of the game are profits, utility or money. It is expressed by payoff functions.

We will introduce the basic assumptions and basic concepts of game theory. Classifications include games (cooperative and non-cooperative games), games with mixed and pure strategies, and the most important concept in game economic theory involving conflicting interests. Usually, the game consists of several players who have to choose between different strategies, which affect the game's gain or benefit. The primary goal is not to defeat fellow players but to maximize the player's own (expected) referred to as zero-sum games.

Regardless of what others do, any player can act with their best behaviour. However, the repetition of some behaviours of others is in some ways related to behavioural outcomes. Thus, the most important aspect of game theory is that the choice between alternative behaviour depends largely on what others are doing [2].

Therefore, game theory is concerned with analysing all conditions in the presence of all possible strategies, but one strategy improves the private benefit of the individual, in game theory it is not enough to consider your own strategies, but the player must also anticipate the best strategies for the opponent.

Typical classic games are used to predict the outcome of a variety of strategies consisting of a limited number of players (or agents) seeking to maximize some individual goal.