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Cairo University
Faculty of Economics and Political Science
Department of Statistics

On the Selection of Prior Distributions in Bayesian Analysis

A Thesis Submitted to the Faculty of Economics and Political Science in Partial Fulfillment of the Requirements for the Degree of Master in Statistics

Presented by
Niveen Ibrahim Aly El-Zayat

Supervisors

Dr. Laila O. El-Zeini
Associate Professor
Department of Statistics
Faculty of Economics and Political Science

Dr. Emad E. A. Soliman
Assistant Professor
Department of Statistics
Faculty of Economics and Political Science

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Name: Niveen Ibrahim Aly El Zayat

Nationality: Egyptian

Date and Place of Birth: 3/9/1973 – Agouza – El Giza

Degree: M.Sc. in Statistics

Specialization: Statistics

Supervisors:

Assoc. Prof. Laila O. El-Zeini

Assis. Prof. Emad E. A. Soliman

Department of Statistics

Faculty of Economics and Political Science

Title of Thesis: On the Selection of Prior Distributions in Bayesian Analysis

Summary: The main objective of the current thesis is to review the best known approaches of selecting the prior distributions of unknown parameters in Bayesian analysis. Two main approaches are available in the literature, namely the noninformative prior approach and the informative prior approach. The thesis throws light on the well-known methodologies of each type as introduced in the literature. Moreover, the study considers the definition, motivations, philosophy and derivation of each type. Furthermore, the thesis discusses their merits and drawbacks. The affinities and distinctions of different types are revealed as well. Applications of the prior distributions of both approaches are demonstrated to get the posterior analysis of some well-known models in econometrics and time series analysis; such as general linear model and autoregressive model of order one. A numerical example is introduced, based on simulation studies for AR(1) model, to compare the performance of the studied priors using some criteria. The results of the comparative study suggest that there is no clear-cut prior distribution recommended for usage where elicitation of suitable prior distribution is based on the time series length and properties. Finally, all priors are demonstrated by some real life time series data sets to illustrate the behavior of the candidate priors.

Abstract

The main objective of the current thesis is to review the best known approaches of selecting the prior distributions of unknown parameters in Bayesian analysis. Two main approaches are available in the literature, namely the noninformative prior approach and the informative prior approach. The thesis throws light on the well-known methodologies of each type as introduced in the literature. Moreover, the study considers the definition, motivations, philosophy and derivation of each type. Furthermore, the thesis discusses their merits and drawbacks. The affinities and distinctions of different types are revealed as well. Applications of the prior distributions of both approaches are demonstrated to get the posterior analysis of some well-known models in econometrics and time series analysis; such as general linear model and autoregressive model of order one. A numerical example is introduced, based on simulation studies for AR(1) model, to compare the performance of the studied priors using some criteria. The results of the comparative study suggest that there is no clear-cut prior distribution recommended for usage where elicitation of suitable prior distribution is based on the time series length and properties. Finally, all priors are demonstrated by some real life time series data sets to illustrate the behavior of the candidate priors.

Key Words: Bayesian analysis - Prior distribution - Posterior distribution - Noninformative prior distributions - Jeffreys' prior – Invariance - Locally uniform prior - Data translated likelihood - Maximal data information prior - Informative prior distributions - Natural conjugate prior - G-prior - General linear model - Autoregressive models of order one.

Supervisors:

Assoc. Prof. Laila O. El-Zeini

Assis. Prof. Emad E. A. Soliman

Department of Statistics

Faculty of Economics and Political Science

Abstract

Prior selection is considered as a crucial difficulty that have ever encountered Bayesian framework for many applications, since prior specification is the prime step to perform a Bayesian analysis to the unknown parameters for the decision making. Bayesian machine updates the prior information available about parameters through the prior distribution in the light of information provided by the likelihood function to get finally the so-called posterior distribution. This last distribution contains all possible information about parameters. Thus, it is used for making inference about the parameters.

That essential rule of prior selection in the structure of Bayesian analysis explains the vast literature in prior selection problem. This selection can be done using one of two main approaches, noninformative prior and informative prior according to the existence of prior information about the parameters in the model of interest.

Noninformative prior approaches are used when no or few information are available about parameters. These approaches are widely accepted in literature since they do not require subjective determinations. Besides, they introduce automatic consecutive steps to derive the posterior results. One of the most well-known noninformative prior is the Jeffreys' prior. Such prior has gained widespread acceptance in many fields due to its simplicity. One of its main features is the invariance property. However, Jeffreys' prior can not be applied in some cases when there are different types of parameters or when no regularity conditions are available. That motivates authors to develop some other noninformative prior distributions. These approaches differ in their philosophies. One of those outstanding approaches is the locally uniform prior proposed by Box and Tiao (1973) that is based on the concept of the data translated likelihood. Another one developed by Zellner (1977), is the maximal data information prior that is based on maximizing the data provided by the sample. That last approach requires developing some informational criteria.

On the other hand, the informative prior distributions are used when information are available about the unknown parameters. Many approaches were developed in literature to quantify such information in a form of probability distribution. The progress in computing facilities motivates authors to develop more accepted informative prior distributions. One of the most famous informative prior approaches is the natural conjugate prior developed by Raiffa and Schlaifer (1961). This prior is chosen such that it has the same functional form as the likelihood function when the last is expressed as a function of the parameters. The only difficulty encounters that type is the specification of hyperparameters. However, there are many methods developed in the literature to solve such a problem. Another type of informative priors is the g-prior introduced by Zellner (1986) to formulate the Bayesian analysis of the general linear model. This type of informative prior is a special case of the natural conjugate one but with less effort required to assess the hyperparameters, since it only requires estimating the location hyperparameters of the coefficient parameters while the variance-covariance matrix is estimated using the design matrix.

Since the study aims to investigate the different types of noninformative and informative prior distributions, a complete perspective over both approaches is displayed and applications to these approaches have been introduced to produce the posterior analysis of the general linear model (GLM) and the AR(1) models. A comparative study is also demonstrated through simulation devices to investigate the efficiency of the different prior approaches to produce the posterior analysis of AR(1) models. All priors are demonstrated by some real life time series data sets to compare the performance of the candidate priors.

From all what have been introduced in the thesis, the current thesis emphasizes the great importance of the prior selection according to the characteristics of the model of interest and to the sample size as well. Thus, caution must be given to the different situations that may be encountered and it is recommended to examine the appropriate prior since no clear-cut method tells the investigator which is the best prior to be used.

Dedicated To my Mother and the Soul of

my Grandfather and Grandmother

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